



## Correlations, risk and crisis: From physiology to finance

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### ABSTRACT

We study the dynamics of correlation and variance in systems under the load of environmental factors. A universal effect in ensembles of similar systems under the load of similar factors is described: in crisis, typically, even before obvious symptoms of crisis appear, correlation increases, and, at the same time, variance (and volatility) increases too. This effect is supported by many experiments and observations of groups of humans, mice, trees, grassy plants, and on financial time series.

A general approach to the explanation of the effect through dynamics of individual adaptation of similar non-interactive individuals to a similar system of external factors is developed. Qualitatively, this approach follows Selye's idea about adaptation energy.

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### 0. Introduction: sources of ideas and data

In many areas of practice, from physiology to economics, psychology, and engineering we have to analyze the behavior of groups of many similar systems, which are adapting to the same or similar environment. Groups of humans in hard living conditions (Far North city, polar expedition, or a hospital, for example), trees under the influence of anthropogenic air pollution, rats under poisoning, banks in financial crisis, enterprises in recession, and many other situations of that type provide us with plenty of important problems, problems of diagnostics and prediction.

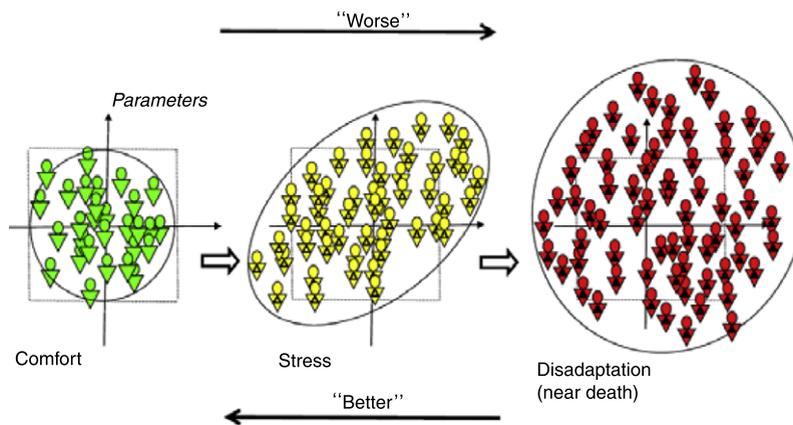
For many such situations, it was found that the correlations between individual systems are better indicators than the value of attributes. More specifically, in thousands of experiments it was shown that in crisis, typically, even before obvious symptoms of crisis appear, the correlations increase, and, at the same time, the variance (volatility) increases too (Fig. 1).

On the other hand, situations with inverse behavior were predicted theoretically and found experimentally [1]. For some systems, it was demonstrated that after the crisis achieves its bottom, it can develop into two directions: recovering (both the correlations and the variance decrease) or fatal catastrophe (the correlations decrease, but the variance continues to increase) (Fig. 1). This makes the problem more intriguing.

If we look only on the state but not on the history then the only difference between comfort and disadaptation in this scheme is the value of variance: in the disadaptation state the variance is larger and the correlations in both cases are low. Qualitatively, the typical behavior of an ensemble of similar systems, which are adapting to the same or similar environment looks as follows:

- In a well-adapted state, the deviations of the systems' state from the average value have relatively low correlations;
- Under increasing of the load of environmental factors some of the systems leave the low correlated comfort cloud and form a low-dimensional highly correlated group (an *order parameter* appears). With further increasing of the load more

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**Fig. 1.** Correlations and variance in crisis. The typical picture:  $Cor \uparrow; Var \uparrow$  – stress;  $Cor \downarrow; Var \downarrow$  – recovering;  $Cor \downarrow; Var \uparrow$  – approaching the disadaptation catastrophe after the bottom of the crisis. In this schematic picture, axes correspond to attributes, normalized to the unite variance in the comfort state.

systems join this highly correlated group. A simplest model based on Selye's ideas about adaptation gives the explanation of this effect (see Section 4.1.2);

- After the load gets over some critical value, the order parameter disappears and the correlations decrease but the variance continues to increase.

There is no proof that this is the only scenario of the changes. Perhaps, it is not. It depends on the choice of parameters, for example. Nevertheless, the first part (appearance of an order parameter) was supported by plenty of experiments and the second part (destroying of the order parameter) is also supported by observation of the systems near death.

Now, after 21 years of studying of this effect [2,3], we maintain that it is universal for groups of similar systems that are sustaining a stress and have an adaptation ability. Hence, a theory at an adequate level of universality is needed.

In this paper we review some data for different kinds of systems: from humans to plants [3–8], and perform also a case study of the thirty largest companies from the British stock market for the period 2006–2008.

In economics, we use also published results of data analysis for equity markets of seven major countries over the period 1960–1990 [9], for the twelve largest European equity markets after the 1987 international equity market crash [10], and for thirty companies from Deutsche Aktienindex (DAX) over the period 1988–1999 [11]. The analysis of correlations is very important for portfolio optimization, and an increase of correlations in a crisis decreases the possibility of risk diversification [12, Chs. 12, 13]. In 1999, it was proposed [13] to use the distance  $d_{ij} = \sqrt{2(1 - \rho_{ij})}$ , where  $\rho_{ij}$  is the correlation coefficient, for the analysis of the hierarchical structure of a market. (This distance for multidimensional time series analysis was analyzed previously in Ref. [14].) The performance of this approach was demonstrated on the stocks used to compute the Dow Jones Industrial Average and on the portfolio of stocks used to compute the S&P 500 index. This approach was further developed and applied (together with more standard correlation analysis) for analysis of anatomy of the Black Monday crisis (October 19, 1987) [15]. In this analysis, hundreds of companies were used.

Stock price changes of the largest 1000 US companies were analyzed for the 2-year period 1994–1995 [16], and statistics of several of the largest eigenvalues of the correlation matrix were evidenced to be far from the random matrix prediction. This kind of analysis was continued for the three major US stock exchanges, namely, the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX), and the National Association of Securities Dealers Automated Quotation (NASDAQ) [17]. Cleaning the correlation matrix by removing the part of the spectrum explainable by random matrix ensembles was proposed [18]. Spectral properties of the correlation matrix were analyzed also for 206 stocks traded in Istanbul Stock Exchange Market during the 5-year period 2000–2005 [19].

Linear and nonlinear co-movements presented in the Real Exchange Rate (RER) in a group of 28 developed and developing countries were studied to clarify the important question about “crisis contagion” [20]: Do strong correlations appear before crisis and provide crisis contagion, or do they grow stronger because of crisis? The spread of the credit crisis (2007–2008) was studied by referring to a correlation network of stocks in the S&P 500 and NASDAQ-100 indices. Current trends demonstrate that the losses in certain markets, follow a cascade or epidemic flow along the correlations of various stocks. But whether or not this idea of epidemic or cascade is a metaphor or a causal model for this crisis is not so obvious [21].

Most of the data, which we collected by ourselves or found in publications, support the hypothesis presented in Fig. 1. In all situations, the definitions of stress and crisis were constructed by experts in specific disciplines on the basis of specific knowledge. What do “better” and “worse” mean? This is a nontrivial special question and from the point of view of very practically oriented researchers the main outcome of modeling may be in the definition of crisis rather than in the explanation of details [22]. In many situations we can detect that one man's crisis is another man's road to prosperity.

Nevertheless, all the experiments are unbiased in the following sense: the definitions of the “better–worse” scale were done before the correlation analysis and did not depend on the results of that analysis. Hence, one can state, that the expert evaluation of the stress and crisis can be (typically) reproduced by the formal analysis of correlations and variance.

The basic model of such generality should include little detail, and we try to make it as simple as possible. We represent the systems, which are adapting to stress, as the systems which optimize distribution of available amount of resource for the neutralization of different harmful factors (we also consider deficit of anything needful as a harmful factor).

The crucial question for these *factor–resource* models is: what is the *resource of adaptation*? This question arose for the first time when Selye published the concept of *adaptation energy* and experimental evidence supporting this idea [23,24]. After that, this notion was significantly improved [25], plenty of indirect evidence supporting this concept was found, but this elusive adaptation energy is still a theoretical concept, and in the modern “Encyclopedia of Stress” we read: “As for adaptation energy, Selye was never able to measure it...” [26]. Nevertheless, the notion of adaptation energy is very useful in the analysis of adaptation and is now in wide use (see, for example, Refs. [27,28]).

The question about the nature of adaptation resource remains important for the economic situation too. The idea of *exchange* helps here: any resource could be exchanged for another one, and the only question is – what is the “exchange rate”, how fast this exchange could be done, what is the margin, how the margin depends on the exchange time, and what is the limit of that exchange. In the zero approximation we can just postulate the universal adaptation resource and hide all the exchange and recovering processes. For biophysics, this exchange idea seems also attractive, but of course there exist some limits on the possible exchange of different resources. Nevertheless, we can follow the Selye arguments and postulate the adaptation energy under the assumption that this is not an “energy”, but just a pool of various exchangeable resources. When an organism achieves the limits of resources exchangeability, the universal non-specific stress and adaptation syndrome transforms (disintegrates) into specific diseases. Near this limit we have to expect the *critical retardation* of exchange processes.

In biophysics, the idea of optimization requires additional explanation. The main source of the optimality idea in biology is the formalization of natural selection and adaptive dynamics. After works of Haldane (1932) [29] and Gause (1934) [30] this direction, with various concepts of *fitness* optimization, was further developed (see, for example, review papers [31–33]). To transfer the evolutionary optimality principles to short and long term adaptation we need the idea of *genocopy–phenocopy interchangeability* [34, p. 117]. The phenotype modifications simulate the optimal genotype, but in a narrower interval of change. We can expect that adaptation also gives the optimal phenotype, but the interval of the possible changes should be even narrower, than for modifications. The idea of convergence of genetic and environmental effects was supported by analysis of genome regulation [35] (the principle of concentration–affinity equivalence). This gives a basis for the optimality assumption in adaptation modeling. For ensembles of man-made systems in economics, the idea of optimality also can be motivated by selection of strategies arguments.

To analyze resource redistribution for the compensation of different environmental factors we have to answer one more question: how is the system of factors organized? Ecology already has a very attractive version for an answer. This is Liebig's Law of the Minimum. The principle behind this law is quite simple. Originally, it meant that the scarcest necessity an organism requires will be the limiting factor to its performance. A bit more generally, the worst factor determines the situation for an organism, and free resource should, perhaps, be assigned for neutralization of that factor (until it loses its leadership).

The opposite principle of factor organization is synergy: the superlinear mutual amplification of factors. Adaptation to Liebig's system of factors, or to any synergistic system, leads to two paradoxes of adaptation:

- *Law of the Minimum paradox* (Section 4.2): If for a randomly selected pair, (State of environment–State of organism), the Law of the Minimum is valid (everything is limited by the factor with the worst value) then, after adaptation, many factors (the maximally possible amount of them) are equally important.
- *Law of the Minimum inverse paradox* (Section 4.3): If for a randomly selected pair, (State of environment–State of organism), many factors are equally important and superlinearly amplify each other then, after adaptation, a smaller amount of factors is important (everything is limited by the factors with the worst non-compensated values, the system approaches the Law of the Minimum).

After introduction of the main ideas and data sources, we are in a position to start more formal consideration.

## 1. Indicators

How can we measure correlations between various attributes in a population? If we have two variables,  $x$  and  $y$ , the answer is simple: we measure  $(x_i, y_i)$  for different individuals ( $i = 1, \dots, n$ ,  $n > 1$  is the number of measurements). The sample correlation coefficient (the Pearson coefficient) is

$$r = \frac{\langle xy \rangle - \langle x \rangle \langle y \rangle}{\sqrt{\langle (x_i - \langle x \rangle)^2 \rangle} \sqrt{\langle (y_i - \langle y \rangle)^2 \rangle}} \quad (1)$$

where  $\langle \dots \rangle$  stands for the sample average value:  $\langle x \rangle = \frac{1}{n} \sum_i x_i$ .

If individuals are characterized by more than two attributes  $\{x^l | l = 1, \dots, m\}$  then we have  $m(m - 1)/2$  correlation coefficients between them,  $r_{jk}$ . In biophysics, we usually analyze correlations between attributes, and each individual organism is represented as a vector of attribute values.

In analysis of financial time series, the standard situation may be considered as a “transposed” one. Each object (stock, enterprise, ...) is represented by a vector of values of a variable (asset return, for example) in a window of time and we study correlations between objects. This is, essentially, just a difference between  $X$  and  $X^T$ , where  $X$  is the matrix of data. In correlation analysis, this difference appears in two operations: centralization (when we subtract means in the computation of covariance) and normalization (when we transform the covariance into the correlation coefficient). In one case, we centralize and normalize the columns of  $X$ : subtract average values in columns, and divide columns on their standard deviations. In another case, we apply these operations to the rows of  $X$ . For financial time series, the synchronous averages and variances (“varieties”) and time averages and variances (“volatilities”) have different statistical properties. This was clearly demonstrated in a special case study [36].

Nevertheless, such a difference does not appear very important for the analysis of the total level of correlations in crisis (just the magnitude of correlation changes, and correlations in time are uniformly less than synchronous ones, this is in agreement with observations from Ref. [36]). More details are presented in the special case study below.

In our case study we demonstrated that in the analysis of financial time series it may be also convenient to study correlations between parameters, not between individuals. It means that we can study correlation between any two time moments and consider data from different individuals as values of random 2D vector. It is necessary to stress that this correlations between two time moments are very different from the standard autocorrelations for stationary time series (which characterize the sample of all pairs of time moments with a given lag in time).

For example, let  $X_{it}$  be a log-return value for  $i$ th stock at time moment  $t$  ( $i = 1, \dots, n, t = \tau + 1, \dots, \tau + T$ ). Each row of the data matrix  $X_{it}$  corresponds to an individual stock and each column corresponds to a time moment. If we normalize and centralize data in rows and calculate the correlation coefficients between rows ( $r_{ij} = \sum_t X_{it}X_{jt}$  for centralized and normalized data) then we find the correlations between stocks. If we normalize and centralize data in columns and calculate the correlation coefficients between them ( $r_{t_1t_2} = \sum_i X_{it_1}X_{it_2}$  for centralized and normalized data) then we find the correlations between time moments. In crisis, dynamics of the correlations between stocks is similar to behavior of the correlations between time moments. One benefit from use of the correlations between time moments is absence of averaging in time (locality): this correlation coefficient depends on data at two time moments. This allows to analyze the anatomy of crisis in time.

To collect information about correlations between many attributes in one indicator, it is possible to use various approaches. First of all, we can evaluate the non-diagonal part of the correlation matrix in any norm, for example, in  $L_p$  norm

$$\|r\|_p = \left( \sum_{j>k} |r_{jk}|^p \right)^{\frac{1}{p}}. \quad (2)$$

If one would like to study strong correlations, then it may be better to delete terms with values below a threshold  $\alpha > 0$  from this sum:

$$G_{p,\alpha} = \left( \sum_{j>k, |r_{jk}|>\alpha} |r_{jk}|^p \right)^{\frac{1}{p}}. \quad (3)$$

This quantity  $G_{p,\alpha}$  is a  $p$ -weight of the  $\alpha$ -correlation graph. The vertices of this graph correspond to variables, and these vertices are connected by edges, if the absolute value of the correspondent sample correlation coefficient exceeds  $\alpha$ :  $|r_{jk}| > \alpha$ . In practice, the most used indicator is the weight  $G = G_{1,0.5}$ , which corresponds to  $p = 1$  and  $\alpha = 0.5$ .

The correlation graphs are used during decades for visualization and analysis of correlations (see, for example, [2,37,38]). Recently, the applications of this approach is intensively developing in data mining [39–41] and econophysics [42,43].

Another group of indicators is produced from the principal components of the data. The principal components are eigenvectors of the covariance matrix and depend on the scales. Under normalization of scales to unit variance, we deal with the correlation matrix. Let  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_m \geq 0$  be eigenvalues of the correlation matrix. In this paper, we use the eigenvalues and eigenvectors of the correlation matrix. It is obvious that  $\langle \lambda \rangle = 1$  and  $m^{p-1} \geq \langle \lambda^p \rangle \geq 1$  for  $p > 1$ ,  $\langle \lambda^p \rangle = 1$  if all non-diagonal correlation coefficients are zero and  $\langle \lambda^p \rangle = m^{p-1}$  if all correlation coefficients are  $\pm 1$ . To select the *dominant* part of principal components, it is necessary to separate the “random” part from the “non-random” part of them. This separation was widely discussed (see, for example, the expository review [44]).

The simplest decision gives *Kaiser’s significance rule*: the significant eigenvalues are those, which are greater than the average value:  $\lambda_i > \langle \lambda \rangle$ . For the eigenvalues of the correlation matrix which we study here, it means  $\lambda_i > 1$ . This rule works reasonably well, when there are several eigenvalues significantly greater than one and the others are smaller, but for a matrix which is close to a random matrix the performance may not be so good. In such cases this method overestimates the number of principal components.

In econophysics, another simple criterion for selection of dominant eigenvalues has become popular [45,17–19]. Let us imagine that the dimension of the data vector  $m$  is large and the amount of data points  $n$  is also large, but their ratio  $q = n/m$  is not. This is the typical situation when we analyze data about thousands of stocks: in this case the time window could not be much larger than the dimension of data vector. Let us compare our analysis of real correlations to the fictitious correlations, which appear in  $m \times n$  data matrices with independent, centralized, normalized and Gaussian matrix elements. The distribution of the sample covariance matrix is the Wishart distribution [46]. If  $n \rightarrow \infty$  for given  $m$  then those fictitious correlations disappear, but if both  $m, n \rightarrow \infty$  for constant  $q > 1$  then there exists the limit distribution of eigenvalues  $\lambda$  with density

$$\rho(\lambda) = \frac{q}{2\pi} \sqrt{\left(\frac{\lambda_{\max}}{\lambda} - 1\right)\left(1 - \frac{\lambda_{\min}}{\lambda}\right)}; \quad \lambda_{\min} \leq \lambda \leq \lambda_{\max}; \tag{4}$$

$$\lambda_{\max/\min} = 1 + \frac{1}{q} \pm 2\sqrt{\frac{1}{q}}.$$

If the amount of points is less than dimension of data, ( $q < 1$ ) the same formula with substitution of  $q$  by  $1/q$  is valid for distribution of non-zero eigenvalues.

Instead of Kaiser’s rule for dominant eigenvalues of the correlation matrix we get  $\lambda_i > \lambda_{\max}$  with  $\lambda_{\max}$  given by Eq. (4). If  $q$  grows to  $\infty$ , this new rule turns back into Kaiser’s rule. If  $q$  is minimal ( $q = 1$ ), then the proposed change of Kaiser’s rule is maximal,  $\lambda_{\max} = 4$  and for dominant eigenvalues of the correlation matrix it should be  $\lambda_i > 4$ . This new estimate is just an analogue of Kaiser’s rule in the case when the amount of data vectors is compatible with the dimension of the data space, and, therefore, the data set is far from the law of large numbers conditions.

Another popular criterion for the selection of dominant eigenvalues gives the so-called *broken stick model*. Consider the closed interval  $J = [0, 1]$ . Suppose  $J$  is partitioned into  $m$  subintervals by randomly selecting  $m - 1$  points from a uniform distribution in the same interval. Denote by  $l_k$  the length of the  $k$ th subinterval in the descending order. Then the expected value of  $l_k$  is

$$\mathbf{E}(l_k) = \frac{1}{m} \sum_{j=k}^m \frac{1}{j}. \tag{5}$$

Following the broken stick model, we have to include into the dominant part those eigenvalues  $\lambda_k$  (principal components), for which

$$\frac{\lambda_1}{\sum_i \lambda_i} > \mathbf{E}(l_1) \ \& \ \frac{\lambda_2}{\sum_i \lambda_i} > \mathbf{E}(l_2) \ \& \ \dots \ \& \ \frac{\lambda_k}{\sum_i \lambda_i} > \mathbf{E}(l_k). \tag{6}$$

If the amount of data vectors  $n$  is less than the data dimension  $m$ , then  $m - n$  eigenvalues are zeros, and in Eqs. (5) and (6) one should take  $n$  subintervals instead of  $m$  ones.

It is worth mentioning that the trace of the correlation matrix is  $m$ , and the broken stick model transforms (for  $m > n$ ) into  $\lambda_i > \sum_{j=i}^m \frac{1}{j}$  ( $i = 1, \dots, k$ ). From the practical point of view, this method slightly underestimates the number of dominant eigenvalues. There are other methods based on the random matrices ensembles, but nobody knows the exact dimension of the empirical data, and the broken stick model works satisfactorily and remains “the method of choice”.

To compare the broken stick model to Kaiser’s rule, let us mention that the first principal component is always significant due to Kaiser’s rule (if there exists at least one nonzero non-diagonal correlation coefficient), but in the broken stick model it needs to be sufficiently large: the inequality  $\lambda_1 > \sum_{j=1}^m \frac{1}{j}$  should hold. In a high dimension  $m$  we can approximate the sum by the quadrature:  $\lambda_1 \gtrsim \ln m$ .

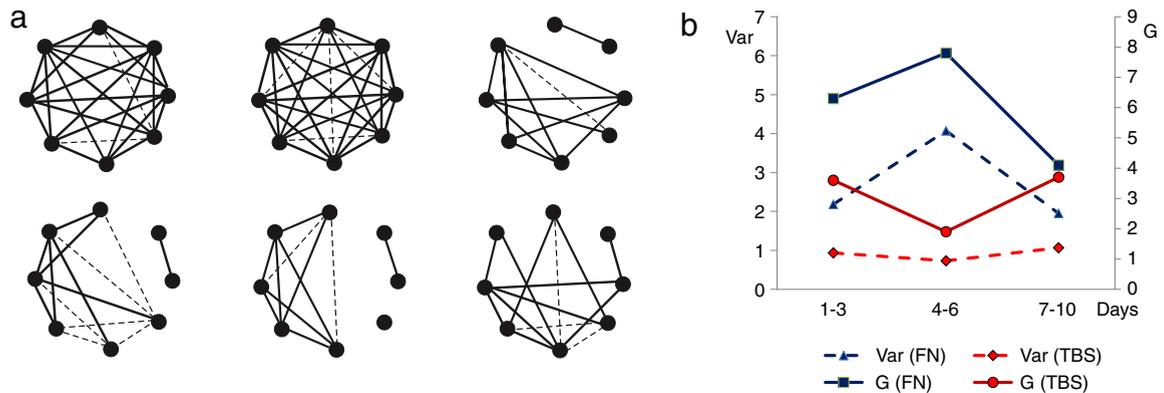
If we have the dominant eigenvalues,  $\lambda_1 \geq \lambda_2 \geq \dots \lambda_l > 0, l < m$ , then we can produce some other measures of the sample correlation:

$$\frac{\lambda_1}{\lambda_l}; \quad \sum_{j=1}^{l-1} \frac{\lambda_j}{\lambda_{j+1}}; \quad \frac{1}{m} \sum_{j=1}^l \lambda_j. \tag{7}$$

Together with  $(\lambda^p)$  ( $p > 1$ , the usual choice is  $p = 2$ ) this system of indicators can be used for an analysis of empirical correlations.

Recently [47] eigenvalues and eigenvectors of the matrix of the absolute values of the correlation coefficients were used for analysis of the New York Stock Exchange (NYSE) traded stocks. The transformation from the correlation matrix to the matrix of absolute values was justified by interpreting the absolute values as measures of interaction strength without considering whether the interaction is positive or negative. This approach gives the possibility to apply the classical theory of positive matrices as well as the graphical representation of them.

The correlation matrix for financial time series is often positive. Therefore, it is often possible to apply the theory of positive matrices to analysis of correlations in financial time series.



**Fig. 2.** (a) Correlation graphs of lipid metabolism for newborn babies. Vertices correspond to different fractions of lipids, solid lines correspond to correlation coefficient between fractions  $|r_{ij}| \geq 0.5$ , dashed lines correspond to  $0.5 > |r_{ij}| \geq 0.25$ . Upper row – Far North (FN), lower row – the temperate belt of Siberia (TBS). From the left to the right: 1st–3rd days (TBS – 123 and FN – 100 babies), 4th–6th days (TBS – 98 and FN – 99 babies), 7th–10th days (TBS – 35 and FN – 29 babies). (b) The weight of the correlation graphs (solid lines) and the variance (dashed lines) for these groups.

The choice of possible indicators is very rich, but happily, many case studies have shown that in analysis of adaptation the simplest weight  $G$  of the correlation graph performs well (better or not worse than all other indicators – see the case study below). Similarity of behavior of various indicators, from simple weight of the correlation graphs to more sophisticated characteristics based on the principal component analysis is expected. Nevertheless, it is desirable to supplement the case studies by comparisons of behavior of different indicators (for example, by scattering plots, correlation analysis or other statistical tools). In our case study (Section 3) we demonstrate that the indicators behave similarly, indeed.

A similar observation was made in Ref. [15]. There the “asset tree” was studied, that is the recently introduced minimum spanning tree description of correlations between stocks. The mean of the normalized dynamic asset tree lengths was considered as a promising indicator of the market dynamics. It appeared that a simple average correlation coefficient gives the same signal in time, as a more sophisticated indicator, the mean of the normalized dynamic asset tree lengths (compare Figs. 1 and 2 from Ref. [15]). In Fig. 12 from that paper very similar behavior of the mean correlation coefficient, the normalized tree length, and the risk of the minimum risk portfolio, as functions of time, was demonstrated.

In many publications in econophysics the average correlation coefficient is used instead of the sums of absolute values in Eq. (3). This is possible because in many financial applications the orientation of the scales is fixed and the difference between positive and negative correlations is very important, for example, for portfolio optimization. In a more general situation we have to use absolute values because we cannot coordinate a priori the direction of different axes.

## 2. Correlation and risk in physiology

Effect of the simultaneous increase of the correlation and variance under stress is supported by series of data from human physiology and medicine. In this section we describe in brief several typical examples. This is a review of already published experimental work. More details are presented in an extended e-print [48] and in original works.

### 2.1. Data from human physiology

The first physiological system we studied in 1980s was the lipid metabolism of healthy newborn babies born in the temperate belt of Siberia (the comfort zone) and in the migrant families of the same ethnic origin in a Far North city.<sup>1</sup> The blood analysis was taken in the morning, on an empty stomach, at the same time each day. All the data were collected during the summer. Eight lipid fractions were analyzed [2]. The resulting correlation graphs are presented in Fig. 2a. Here solid lines represent the correlation coefficient  $|r_{ij}| \geq 0.5$ , dashed lines represent correlation coefficient  $0.5 > |r_{ij}| \geq 0.25$ . Variance monotonically increases with the weight of the correlation graph (Fig. 2b).

Many other systems were studied. We analyzed the activity of enzymes in human leukocytes during the short-term adaptation (20 days) of groups of healthy 20–30 year old men who change their climate zone [49,50]:

- From the temperate belt of Siberia (Krasnoyarsk, comfort zone) to Far North in summer and in winter;
- From Far North to the South resort (Sochi, Black Sea) in summer;
- From the temperate belt of Russia to the South resort (Sochi, Black Sea) in summer.

<sup>1</sup> The parents lived there in standard city conditions.

This analysis supports the basic hypothesis and, on the other hand, could be used for prediction of the most dangerous periods in adaptation, which need special care.

We selected the group of 54 people who moved to Far North, that had any illness during the period of short-term adaptation. After 6 months at Far North, this test group demonstrates much higher correlations between activity of enzymes than the control group (98 people without illness during the adaptation period). We analyzed the activity of enzymes (alkaline phosphatase, acid phosphatase, succinate dehydrogenase, glyceraldehyde-3-phosphate dehydrogenase, glycerol-3-phosphate dehydrogenase, and glucose-6-phosphate dehydrogenase) in leucocytes:  $G = 5.81$  in the test group versus  $G = 1.36$  in the control group. To compare the dimensionless variance for these groups, we normalize the activity of enzymes to unite sample means (it is senseless to use the trace of the covariance matrix without normalization because normal activities of enzymes differ in order of magnitude). For the test group, the sum of the enzyme variances is 0.388, and for the control group it is 1.204.

Obesity is a serious problem of contemporary medicine in developed countries. The study was conducted on patients (more than 70 people) with different levels of obesity [6]. The patients were divided into three groups by the level of disease. Database with 50 attributes was studied (blood morphology, cholesterol level including fractions, creatinine, urea).

During 30 days patients received a standard treatment consisting of a diet, physical activity, pharmacological treatment, physical therapy and hydrotherapy. It was shown that the weight of the correlation graph  $G$  of more informative parameters was originally high and monotonically dependent on the level of sickness. It decreased during therapy.

## 2.2. Data from ecological physiology of plants

The effect (Fig. 1) exists for plants too. It was demonstrated, for example, by analysis of the impact of emissions from a heat power station on Scots pine [51]. For diagnostic purposes the secondary metabolites of phenolic nature were used. They are much more stable than the primary products and hold the information about the past impact of environment on the plant organism for a longer time.

The test group consisted of 10 Scots pines (*Pinus sylvestric* L) in a 40 year old stand of the II class in the emission tongue 10 km from the power station. The station had been operating on brown coal for 45 years. The control group of 10 Scots pines was from a stand of the same age and forest type, growing outside the industrial emission area. The needles for analysis were one year old from the shoots in the middle part of the crown. The samples were taken in spring in bud swelling period. The individual composition of the alcohol extract of needles was studied by high efficiency liquid chromatography. 26 individual phenolic compounds were identified for all samples and used in the analysis.

No reliable difference was found in the test group and control group average compositions. For example, the results for Proanthocyanidin content (mg/g dry weight) were as follows:

- Total  $37.4 \pm 3.2$  (test) versus  $36.8 \pm 2.0$  (control).

Nevertheless, the sample variance in the test group was 2.56 times higher, and the difference in the correlations was huge:  $G = 17.29$  for the test group versus  $G = 3.79$  in the control group.

The grassy plants under trampling load also demonstrate a similar effect [8]. The grassy plants in an oak forests are studied. For analysis, fragments of forests were selected, where the densities of trees and bushes were the same. The difference between these fragments was in damage to the soil surface by trampling. The studied physiological attributes were: the height of sprouts, the length of roots, the diameter of roots, the amount of roots, the area of leaves, the area of roots. Again, the weight of the correlation graph and the variance monotonically increased with the trampling load.

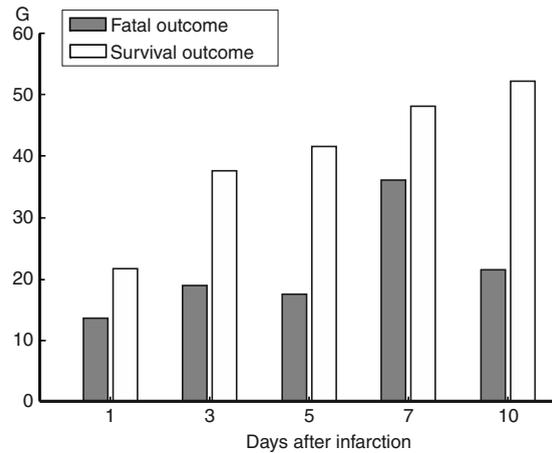
## 2.3. The problem of “no return” points

It is practically important to understand where the system is going: (i) to the bottom of the crisis with the possibility to recover after that bottom, (ii) to the normal state, from the bottom, or (iii) to the “no return” point, after which it cannot recover.

Situations between the comfort zone and the crisis has been studied for dozens' of systems, and the main effect is supported by much empirical evidence. The situation near the “no return” point is much less studied. Nevertheless, some observations support the hypothesis presented for this case in Fig. 1: when approaching the fatal situation correlations decrease and variance increases.

This problem was studied with the analysis of fatal outcomes in oncological [52] and cardiological [53] clinics, and also in special experiments with acute hemolytic anemia caused by phenylhydrazine in mice [7]. The main result is: when approaching the no-return point, correlations disappear ( $G$  decreases), and variance typically does continue to increase.

For example, the dynamics of correlations between physiological parameters after myocardial infarction was studied in Ref. [53]. For each patient (more than 100 people), three groups of parameters were measured: echocardiography-derived variables (end-systolic and end-diastolic indexes, stroke index, and ejection fraction), parameters of central hemodynamics (systolic and diastolic arterial pressure, stroke volume, heart rate, the minute circulation volume, and specific peripheral resistance), biochemical parameters (lactate dehydrogenase, the heart isoenzyme of lactate dehydrogenase LDH1, aspartate transaminase, and alanine transaminase), and also leucocytes. Two groups were analyzed after 10 days of monitoring: the



**Fig. 3.** Dynamics of weight of the correlation graphs of echocardiography-derived variables, parameters of central hemodynamics, biochemical parameters, and also leucocytes during 10 days after myocardial infarction for two groups of patients: for the survival outcome and for the fatal outcome. Here  $G$  is the sum of the strongest correlations  $|r_{ij}| > 0.4, i \neq j$  [53].

patients with a lethal outcome, and the patients with a survival outcome (with compatible amounts of group members). These groups do not differ significantly in the average values of parameters and are not separable in the space measured attributes. Nevertheless, the dynamics of the correlations in the groups are essentially different. For the fatal outcome correlations were stably low (with a short pulse at the 7th day), for the survival outcome, the correlations were higher and monotonically grew. This growth can be interpreted as return to the “normal crisis” (the central position in Fig. 1).

Topologically, the correlation graph for the survival outcome included two persistent triangles with strong correlations: the central hemodynamics triangle, minute circulation volume – stroke volume – specific peripheral resistance, and the heart hemodynamics triangle, specific peripheral resistance – stroke index – end-diastolic indexes. The group with a fatal outcome had no such persistent triangles in the correlation graph.

In the analysis of fatal outcomes for oncological patients and in special experiments with acute hemolytic anemia caused by phenylhydrazine in mice one more effect was observed: for a short time before death the correlations increased, and then fell down (see also the pulse in Fig. 3). This short pulse of the correlations (in our observations, usually for one day, a day which precedes the fatal outcome) is opposite to the major trend of the systems in their approach to death. We cannot claim universality of this effect and it requires additional empirical study.

### 3. Correlations and risk in economics. Empirical data

#### 3.1. Thirty companies from the FTSE 100 index: A case study

##### 3.1.1. Data and indicators

For the analysis of correlations in financial systems we used the daily closing values over the time period 03.01.2006–20.11.2008 for companies that are registered in the FTSE 100 index (Financial Times Stock Exchange Index). The FTSE 100 is a market-capitalization weighted index representing the performance of the 100 largest UK-domiciled blue chip companies which pass screening for size and liquidity. The index represents approximately 88.03% of the UKs market capitalization. FTSE 100 constituents are all traded on the London Stock Exchanges SETS trading system. We selected 30 companies that had the highest value of the capital (on the 1st of January 2007) and stand for different types of business as well. The list of the companies and business types is displayed in Table 1.

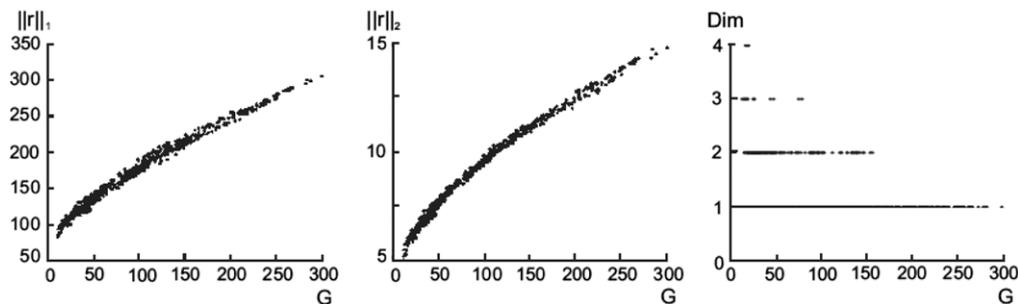
Data for these companies are available from the Yahoo!Finance web-site. For data cleaning we use also information for the selected period available at the London Stock Exchange web-site. Let  $x_i(t)$  denote the closing stock price for the  $i$ th company at the moment  $t$ , where  $i = \overline{1, 30}, t = \overline{1, 732}$ . We analyze the correlations of logarithmic returns:  $x'_i(t) = \ln \frac{x_i(t)}{x_i(t-1)}$ ,  $t = \overline{2, 732}$  in sliding time windows of length  $p = 20$ , this corresponds approximately to 4 weeks of 5 trading days,  $t = p + 1, 732$ . The correlation coefficients  $r_{ij}(t)$  and all indicators for time moment  $t$  are calculated in the time window  $[t - p, t - 1]$ , which precedes  $t$ . This is important if we would like to consider changes in these indicators as precursors of crisis.

The information about the level of correlations could be represented in several ways. Here we compare 4 indicators:

- The non-diagonal part of the correlation matrix in  $L_2$  norm –  $\|r\|_2$ ;
- The non-diagonal part of the correlation matrix in  $L_1$  norm –  $\|r\|_1$ ;
- The sum of the strongest elements  $G = \sum_{j>i, |r_{ij}|>0.5} |r_{ij}|$ ;
- The amount Dim of principal components estimated due to the broken stick model.

**Table 1**  
Thirty largest companies for analysis from the FTSE 100 index.

Number	Business type	Company	Abbreviation
1	Mining	Anglo American plc	AAL
2		BHP billiton	BHP
3	Energy (oil/gas)	BG group	BG
4		BP	BP
5		Royal Dutch shell	RDSB
6	Energy (distribution)	Centrica	CNA
7		National grid	NG
8	Finance (bank)	Barclays plc	BARC
9		HBOS	HBOS
10		HSBC HLDG	HSBC
11		Lloyds	LLOY
12	Finance (insurance)	Admiral	ADM
13		Aviva	AV
14		LandSecurities	LAND
15		Prudential	PRU
16		Standard chartered	STAN
17	Food production	Unilever	ULVR
18	Consumer Goods/food/drinks	Diageo	DGE
19		SABMiller	SAB
20		TESCO	TSCO
21	Tobacco	British American tobacco	BATS
22		Imperial tobacco	IMT
23	Pharmaceuticals (inc. research)	AstraZeneca	AZN
24		GlaxoSmithKline	GSK
25	Telecommunications	BT group	BTA
26		Vodafone	VOD
27	Travel/leisure	Compass group	CPG
28	Media (broadcasting)	British sky broadcasting	BSY
29	Aerospace/ Defence	BAE system	BA
30		Rolls-Royce	RR



**Fig. 4.** Scatter diagrams for three pairs of indicators:  $G - \|r\|_1$ ,  $G - \|r\|_2$ , and  $G - \text{Dim}$ , where  $\text{Dim}$  is amount of principal components estimated due to the broken stick model.

The dynamics of the first three indicators are quite similar. Scatter diagrams (Fig. 4) demonstrate a strong connection between the indicators. We used the weight of the correlation graph  $G$  (the sum of the strongest correlations  $r_{ij} > 0.5$ ,  $i \neq j$ ) for our further analysis.

Fig. 5 allows us to compare dynamics of correlation, dimension and variance to the value of FTSE100. Correlations increase when the market goes down and decrease when it recovers. Dynamics of variance of log-returns has the same tendency. To analyze the critical periods in more detail, let us select several time intervals and several positions of the sliding window inside these intervals.

### 3.1.2. Correlation graphs for companies

We extracted 2 intervals for more detailed analysis. The first interval, 10/04/2006–21/07/2006, represents the FTSE index decrease and restoration in spring and summer 2006. The second interval, 02/06/2008–04/11/2008, is a part of the financial crisis. In each interval we selected six points and analyzed the structure of correlations for each of these points (for a time

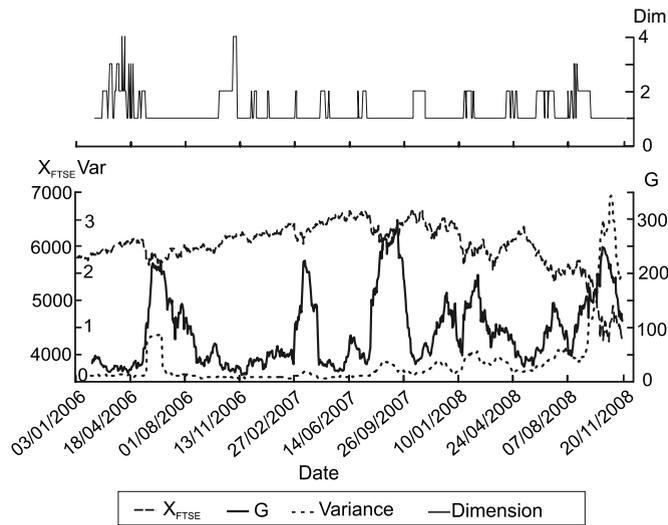


Fig. 5. Dynamics of FTSE index,  $G$ , Variance, and Dimension estimated due to the broken stick model.

window, which precedes this point). For each selected point, we create a correlation graph, solid lines represent correlation coefficient  $|r_{ij}| \geq \sqrt{0.5}$  ( $\sqrt{0.5} = \cos(\pi/4) \approx 0.707$ ), dashed lines represent correlation coefficient  $\sqrt{0.5} > |r_{ij}| \geq 0.5$ : (Figs. 6c, d, 7c, d). On these correlation graphs it is easy to observe, how critical correlations appear, how are they distributed between different sectors of economics, and how the crisis moves from one sector to another.

There is no symmetry between the periods of the FTSE index decrease and recovering. For example, in Fig. 6c we see that at the beginning (falling down) the correlations inside the financial sector are important and some correlations inside industry are also high, but in the corresponding recovering period (Fig. 6d) the correlations between industry and financial institutions become more important.

All the indicators demonstrate the most interesting behavior at the end of 2008 (Fig. 5). The growth of variance in the last peak is extremely high, but the increase of correlations is rather modest. If we follow the logic of the basic hypothesis (Fig. 1), then we should suspect that the system is going to “the other side of crisis”, not to recovery, but to disadaptation, this may be the most dangerous symptom.

### 3.1.3. Graphs for correlations in time

The vector of attributes that represents a company is a 20 day fragment of the time series. In standard biophysical research, we studied correlations between attributes of an individual, and rarely, correlation between individuals for different attributes. In econophysics the standard situation is opposite. Correlation in time is evidenced to be less than correlation between companies [36]. Nevertheless, correlation between days in a given time window may be a good indicator of crisis.

Let us use here  $G_T$  for the weight of the correlation graph in time. Because correlation in time is less than between stocks, we select here another threshold:  $G_T$  is the sum of the correlation coefficients with absolute value greater then 0.25. FTSE dynamics together with values of  $G_T$  are presented in Fig. 8. Solid lines represent a correlation coefficient  $|r_{ij}| \geq 0.5$ , dashed lines represent a correlation coefficient  $0.5 > |r_{ij}| \geq 0.25$ .

On the Figs. 9 and 10 we combined graphs of days correlations – 20 trading days prior to the selected days.

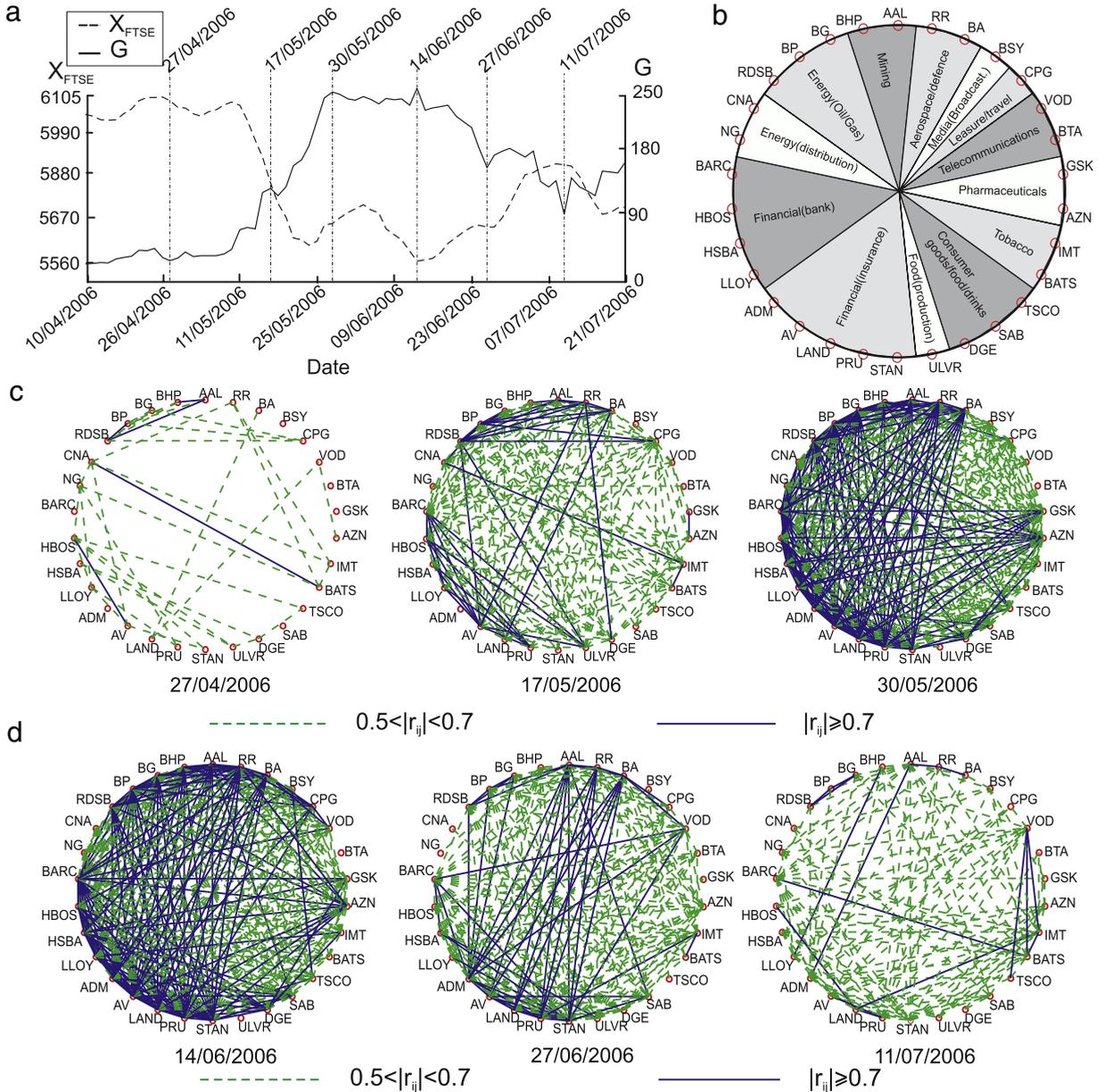
An analysis of the dynamics of  $G_T$  allows us to formulate a hypothesis: typically, after the increase of  $G_T$  the decrease of FTSE100 index follows (and, the decrease of  $G_T$  precedes the increase of FTSE100). The time delay is approximately two working weeks. In that sense, the correlation in time seems to be better indicator of the future change, than the correlation between stocks which has no such time gap. On the other hand, the amplitude of change of  $G_T$  is much smaller, and some of the decreases of the FTSE100 index could not be predicted by increases of  $G_T$  (Fig. 8).

These observations are still preliminary and need future analysis for different financial time series.

A strong correlation between days appears also with some time gap: the links emerge, not usually between nearest days, but mostly with an interval 4–15 days (see Figs. 9 and 10).

## 3.2. Correlations and crisis in financial time series

In economics and finance, the correlation matrix is very important for the practical problem of portfolio optimization and minimization of risk. Hence, an important problem arises: are correlations constant or not? The hypothesis about constant correlations was tested for monthly excess returns for seven countries (Germany, France, UK, Sweden, Japan, Canada, and US) over the period 1960–90 [9]. Correlation matrices were calculated over a sliding window of five years. The inclusion of



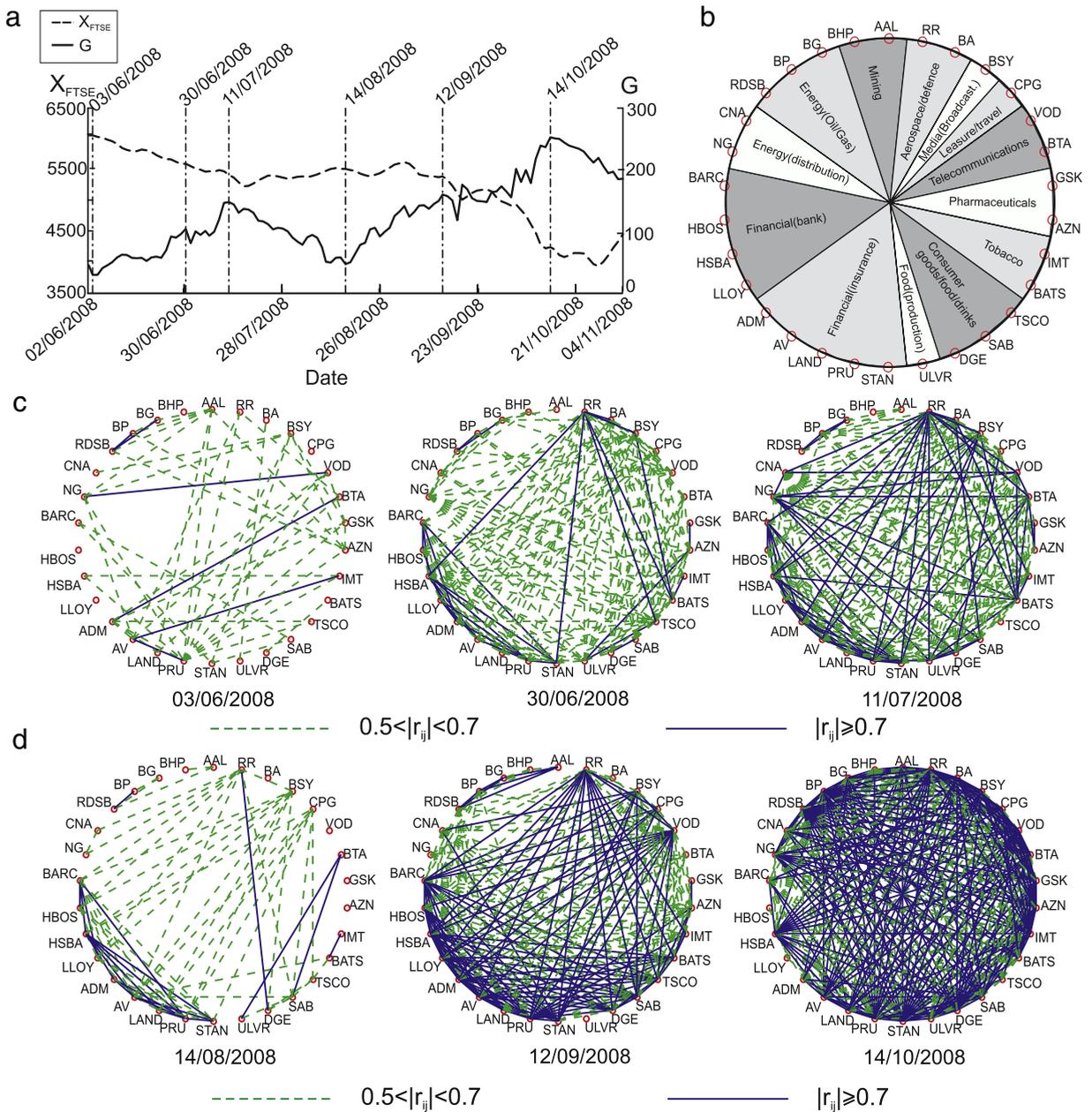
**Fig. 6.** Correlation graphs for six positions of the sliding time window on interval 10/04/2006–21/07/2006. (a) Dynamics of FTSE100 (dashed line) and of  $G$  (solid line) over the interval, vertical lines correspond to the points that were used for the correlation graphs. (b) Thirty companies for analysis and their distributions over various sectors of economics. (c) The correlation graphs for the first three points, FTSE100 decreases, the correlation graph becomes more connective. (d) The correlation graphs for the last three points, FTSE100 increases, the correlation graph becomes less connective.

October 1987 in the window led to an increase of correlation in that window. After an analysis of correlations in six periods of five years the null hypothesis of a constant correlation matrix was rejected. In addition, the *conditional correlation matrix* was studied. The multivariate process for asset return was presented as

$$R_t = m_{t-1} + e_t; \quad m_{t-1} = \mathbf{E}(R_t | F_{t-1}), \tag{8}$$

where  $R_t$  is a vector of asset returns and  $m_{t-1}$  is the vector of expected returns at time  $t$  conditioned on the information set  $F_{t-1}$  from the previous step. Vector  $e_t$  is the unexpected (unpredicted) component in the asset returns. Correlations between its components are called *conditional correlations*. It was demonstrated that these conditional correlations are also not constant. Two types of change were found. Firstly, the correlations have a statistically significant time trend and grow in time. The average increase in correlation over 30 years is 0.36. Secondly, correlations in periods of high volatility (high variance) are higher. To obtain this result, the following model for the correlation coefficient was identified:

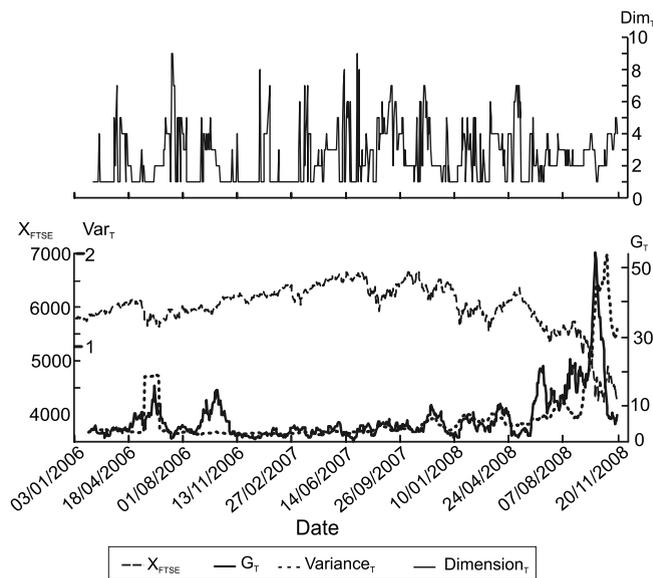
$$r_t^{i,us} = r_0^{i,us} + r_1^{i,us} S_t^{us}, \tag{9}$$



**Fig. 7.** Correlation graphs for six positions of the sliding time window on interval 02/06/2008–04/11/2008. (a) Dynamics of FTSE100 (dashed line) and of  $G$  (solid line) over the interval, vertical lines correspond to the points that were used for the correlation graphs. (b) Thirty companies for analysis and their distributions over various sectors of economics. (c) The correlation graphs for the first three points, FTSE100 decreases, the correlation graph becomes more connective. Between the third and the 4th points FTSE100 increased, and the first graph here is more rarefied than at the third point. Between the third and the 4th points FTSE100 slightly increased, correlation decreased, and the first graph at the next row is more rarefied than at the third point. (d) The correlation graphs for the last three points, FTSE100 decreases, the correlation graph becomes more connective.

where  $r_t^{i,us}$  is the correlation coefficient between the unexpected (unpredicted) components in the asset returns for the  $i$ th country and the US,  $S_t$  is a dummy variable that takes the value 1 if the estimated conditional variance of the US market for time  $t$  is greater than its unconditional (mean) value and 0 otherwise. The estimated coefficient  $r_1$  is positive for all countries. The average over all countries for  $r_0$  is equal to 0.430, while the average turbulence effect  $r_1$  is 0.117 [9]. Finally, it was demonstrated that other informational variables can explain more changes in correlations than just the “high volatility–low volatility” binning.

To analyze correlations between European equity markets before and after October 1987, three 76-month periods were compared: February 1975–May 1981, June 1981–September 1987, and November 1987–February 1994 [10]. The average correlation coefficient for 13 equity markets (Europe + US) increased from 0.37 in June 1981–September 1987 to 0.5 in



**Fig. 8.** Dynamics of the market  $X_{FTSE}$ , weight of correlation  $G_T$  the sum of the correlation coefficients with absolute value greater than 0.25, Variance (volatility), and dimension of the correlation matrix estimated due to the broken stick model.

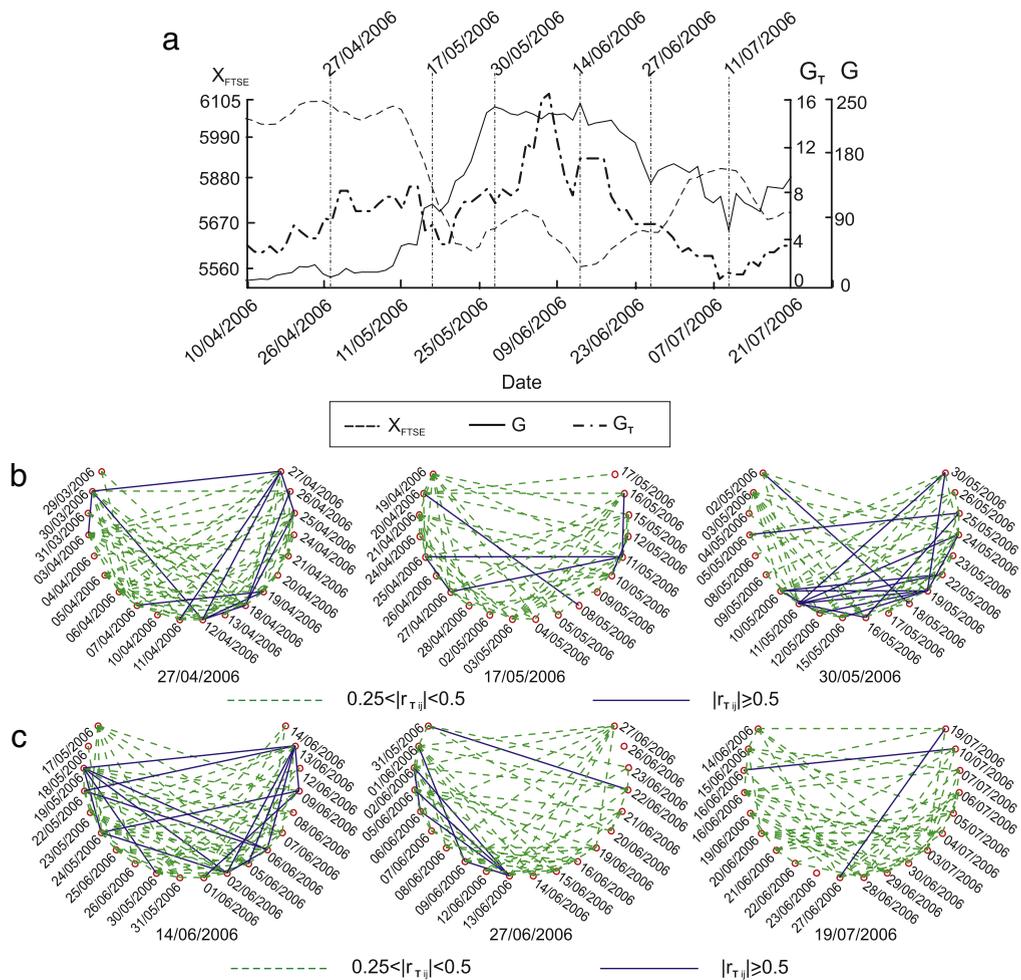
November 1987–February 1994. The amount of significant principal components selected by Kaiser’s rule decreases from 3 (in both periods before October 1987) to 2 (in the period after October 1987) for all markets and even from 3 to 1 for 12 European markets [10]. Of course, in average values for such long periods it is impossible to distinguish the consequences of the October 1987 catastrophe and a trend of correlation coefficients (that is, presumably, nonlinear).

Non-stationarity of the correlation matrix was demonstrated in a detailed study of the financial empirical correlation matrix of the 30 companies which Deutsche Aktienindex (DAX) comprised during the period 1988–1999 [11]. The time interval (time window) is set to 30 and continuously moved over the whole period. It was evidenced that the drawups and the drawdowns of the global index (DAX) are governed, respectively, by dynamics of a significantly distinct nature. The drawdowns are dominated by one strongly collective eigenstate with a large eigenvalue. The opposite applies to drawups: the largest eigenvalue moves down which is compensated by a simultaneous elevation of lower eigenvalues. Distribution of correlation coefficients for these data have a distinctive bell-like shape both for one time window (inside one correlation matrix) and for ensemble of such sliding windows in a long time period.

This observation supports the idea of applying the theory of the Gaussian matrix ensembles to the analysis of financial time series. The random matrix theory gives a framework for analysis of the cross-correlation matrix for multidimensional time series. In that framework, stock price changes of the largest 1000 US companies were analyzed for the 2-year period 1994–1995 [16], and statistics of several of the largest eigenvalues was evidenced to be far from the random matrix prediction, but the distribution of “the rest” of the eigenvalues and eigenvectors satisfies the random matrix ensemble. The crucial question is: where is the border between the random and the non-random parts of spectra? Formula (4) gives in this case  $\lambda_{\max} \approx 2$ . The random matrix theory predicts for the Gaussian orthogonal ensembles that the components of the normalized eigenvectors are distributed according to a Gaussian probability distribution with mean zero and variance one. Eigenvectors corresponding to most eigenvalues in the “bulk” ( $\lambda < 2$ ) have the Gaussian distribution, but eigenvectors with bigger eigenvalues significantly deviate from this [16].

This kind of analysis was continued for the three major US stock exchanges, namely the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX), and the National Association of Securities Dealers Automated Quotation (NASDAQ) [17]. The concept of “deviating eigenvectors” was developed, these vectors correspond to the eigenvalues which are systematically outside the random matrices ensembles predictions. Analysis of “deviating eigenvectors” which are outside the random matrices ensembles predictions (4) gives information of major factors common to all stocks, or to large business sectors. The largest eigenvalue was identified as the “market mode”. During periods of high market volatility values of the largest eigenvalue are large. This fact was commented as a strong collective behavior in regimes of high volatility. For the largest eigenvalue, the distribution of coordinates of the eigenvector has very remarkable properties:

- It is much more uniform than the prediction of the random matrix theory (authors of Ref. [17] described this vector as “approximately uniform”, suggesting that all stocks participate in this “market mode”);
- Almost all components of that eigenvector have the same sign.
- A large degree of cross correlations between stocks can be attributed to the influence of the largest eigenvalue and its corresponding eigenvector.



**Fig. 9.** Graphs for correlation in time for six positions of the sliding time window on interval 10/04/2006–21/07/2006. (a) Dynamics of FTSE100 (dashed line),  $G$  (solid line) and  $G_T$  (dash-and-dot line) over the interval, vertical lines correspond to the points that were used for the correlation graphs. (b) The correlation graphs for the first three points: FTSE100 decreases and the correlation graph becomes more connective. (c) The correlation graphs for the last three points: FTSE100 increases and the correlation graph becomes less connective.

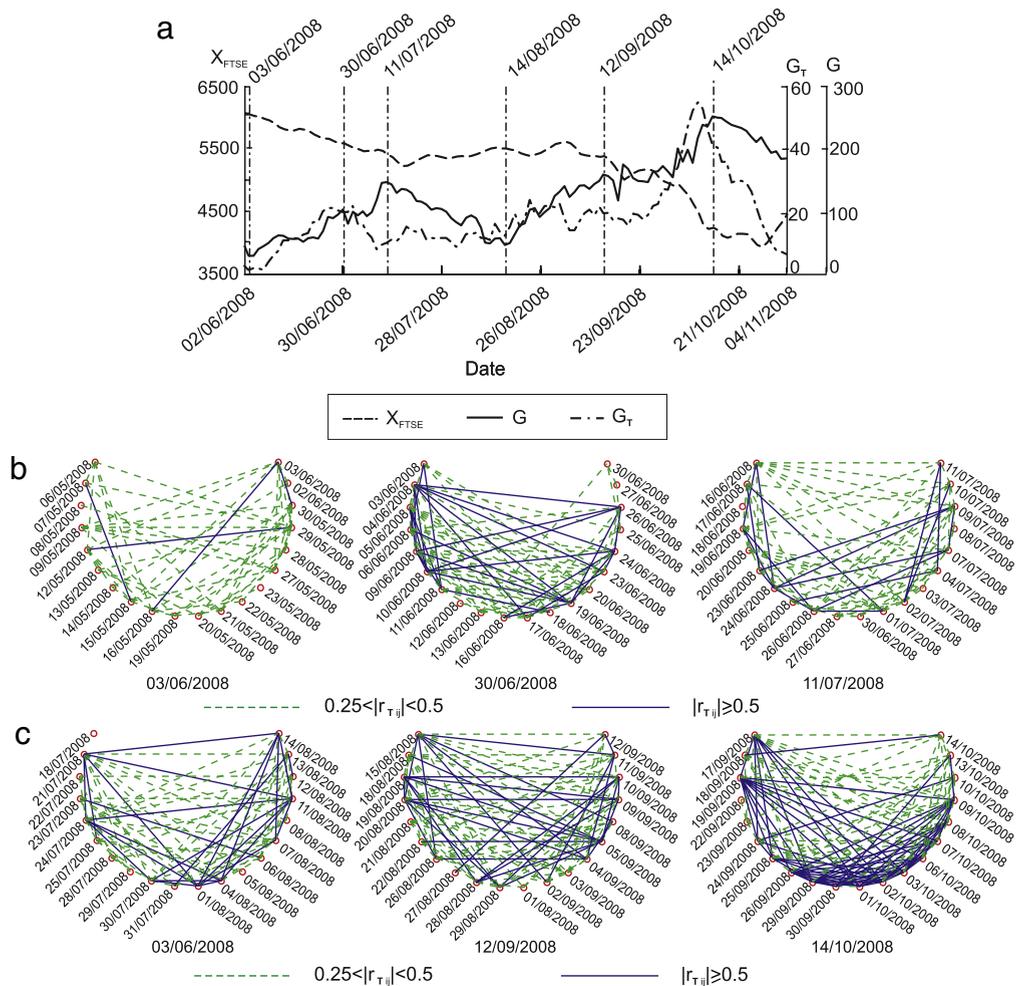
Two interpretations of this eigenvector were proposed: it corresponds either to the common strong factor that affects all stocks, or it represents the “collective response” of the entire market to stimuli.

Spectral properties of the correlation matrix were analyzed also for 206 stocks traded in the Istanbul Stock Exchange Market during the 5-year period 2000–2005 [19]. One of the main results of this research is the observation that the correlations among stocks are mostly positive and tend to increase during crises. The number of significant eigenvalues (outside the random matrix interval) is smaller than it was found in previous study of the well-developed international market in the US. The possible interpretation is: the emerging market is ruling by smaller amount of factors.

An increase of correlations in a time of crisis was demonstrated by the analysis of 150 years of market dynamics [54]. As a result, in the year 2004 it was mentioned very optimistically: “Our tests suggest that the structure of global correlations shifts considerably through time. It is currently near an historical high-approaching levels of correlation last experienced during the Great Depression”. Nevertheless, it remains unclear, does the correlation cause the transmission chain of collapse or is it inextricably tied to it [21]?

There are several types of explanation of these correlation effects. One can look for the specific reasons in the balance between specialization and globalization, in specific fiscal, monetary, legal, cultural or even language conditions, in dynamics of fundamental economic variables such as interest rates and dividend yields, in the portfolio optimization by investors, and in many similar more or less important processes. These specific explanations could work, but for such a general effect it is desirable to find a theory of compatible generality. Now we can mention three sources for such a theory:

- (1) Theory of individual adaptation of similar individuals to a similar system of factors;
- (2) Theory of interaction: information interaction, co-ordination, or deeper integration;
- (3) Theory of collective effects in market dynamics.



**Fig. 10.** Correlation graphs for six positions of the sliding time window on interval 02/06/2008–04/11/2008. (a) Dynamics of FTSE100 (dashed line),  $G$  (solid line) and  $G_T$  (dash-and-dot line) over the interval, vertical lines correspond to the points that were used for the correlation graphs. (b) Thirty companies for analysis and their distributions over various sectors of economics. (c) The correlation graphs for the first three points: FTSE100 decreases and the correlation graph becomes more connective. Between the third and the 4th points FTSE100 increases, and the first graph here is more rarefied than at the third point. Between the third and the 4th points FTSE100 slightly increases, correlation decreased, and the first graph at the next row is more rarefied than at the third point. (d) The correlation graphs for the last three points: FTSE100 decreases and the correlation graph becomes more connective.

The first approach (supported by biological data) is a sort of mean-field theory: everybody is adapting to a field of common factors, and altogether change the state of that system. There are two types of argumentation here: similarity of factors, or similarity of adaptation mechanisms (or both):

- In the period of crisis the same challenges appear for most of the market participants, and correlation increases because they have to answer the same challenge and struggle with the same factors.
- In the period of crisis all participants are under pressure. The nature of that pressure may be different, but the mobilization mechanisms are mostly universal. Similar attempts at adaptation produce correlation as a consequence of crisis.

This theory is focused on the adaptation process, but may be included into any theory of economical dynamics as adaptation feedback. We study the adaptation of individuals in the “mean field”, and consider dynamics of this field as external conditions.

The interaction theory may be much more rich (and complicated). For example, it can consider the following effect of behavior in crisis: there is a lack of information and of known optimal solutions, therefore, different agents try to find clues to rational behavior in the behavior of other agents, and the correlation increases. Coordination in management and in financial politics is an obvious effect of interaction too, and we can observe also a deeper integration, which causes fluxes of moneys and goods.

Collective effects in market dynamics may also generate correlations and, on the other hand, can interact with correlations which appear by any specific or nonspecific reasons. For example, high levels of correlation often lead to the loss of dissipation in dynamics and may cause instability.

Further in this work, we focus on the theory of individual adaptation of similar individuals to a similar system of factors.

## 4. Theoretical approaches

### 4.1. The “energy of adaptation” and factors-resources models

#### 4.1.1. Factors and systems

Let us consider several systems that are under the influence of several factors  $F_1, \dots, F_q$ . Each factor has its intensity  $f_i$  ( $i = 1, \dots, q$ ). For convenience, we consider all these factors as harmful (later, after we introduce fitness  $W$ , it will mean that all partial derivatives are non-positive  $\partial W / \partial f_i \leq 0$ , this is a formal definition of “harm”). This is just a convention about the choice of axes directions: a wholesome factor is just a “minus harmful” factor.

Each system has its adaptation systems, a “shield” that can decrease the influence of these factors. In the simplest case, it means that each system has an available adaptation resource,  $R$ , which can be distributed for neutralization of factors: instead of factor intensities  $f_i$  the system is under pressure from factor values  $f_i - a_i r_i$  (where  $a_i > 0$  is the coefficient of efficiency of factor  $F_i$  neutralization by the adaptation system and  $r_i$  is the share of the adaptation resource assigned for the neutralization of factor  $F_i$ ,  $\sum_i r_i \leq R$ ). The zero value  $f_i - a_i r_i = 0$  is optimal (the fully compensated factor), and further compensation is impossible and senseless.

Interaction of each system with a factor  $F_i$  is described by two quantities: the factor  $F_i$  uncompensated pressure  $\psi_i = f_i - a_i r_i$  and the resource assigned to the factor  $F_i$  neutralization. The question about interaction of various factors is very important, but, first of all, let us study a one-factor model.

#### 4.1.2. The Selye model

Already simple one-factor models support the observed effect of the correlation increase. In these models, observable properties of interest  $x_k$  ( $k = 1, \dots, m$ ) can be modeled as functions of factor pressure  $\psi$  plus some noise  $\epsilon_k$ .

Let us consider one-factor systems and linear functions (the simplest case):

$$x_k = \mu_k + l_k \psi + \epsilon_k, \quad (10)$$

where  $\mu_k$  is the mean value of  $x_k$  for fully compensated factor,  $l_k$  is a coefficient,  $\psi = f - a r_f \geq 0$ , and  $r_f \leq R$  is amount of available resource assigned for the factor neutralization. The values of  $\mu_k$  could be considered as “normal” (in the sense opposite to “pathology”), and noise  $\epsilon_k$  reflects variability of norm. This is not a dynamic equation and describes just one action of resource assignment. If we add time  $t$  then a two-dimensional array appears  $x_{kt}$ .

We can call these models the “tension-driven models” or even the “Selye models” because these models may be extracted from the Selye concept of adaptation energy [23,24] (Selye did not use equations, but qualitatively these models were present in his reasoning).

If systems compensate as much of the factor value, as possible, then  $r_f = \min\{R, f/a\}$ , and we can write:

$$\psi = \begin{cases} f - aR, & \text{if } f > aR; \\ 0, & \text{else.} \end{cases} \quad (11)$$

The *nonlinearity* of the Selye model is in the dependence of  $\psi$  on the factor pressure  $f$ . Already the simple dependence (11) gives the phase transition picture. Individual systems may be different by the value of factor intensity (the local intensity variability), by the amount of available resource  $R$  and, of course, by the random values  $\epsilon_k$ . For small  $f$  all  $\psi = 0$ , all systems are in comfort zone and all the difference between them is in the noise variables  $\epsilon_k$ . In this state, the correlations are defined by the correlations in noise values and are, presumably, low.

With increasing  $f$  the separation appears: some systems remain in the comfort “condensate” ( $\psi = 0$ ), and others already do not have enough resource for a full compensation of the factor load and vary in the value of  $\psi$ . Two fractions appear, a lowly correlated condensate with  $\psi = 0$  and a highly correlated fraction with different values of  $\psi > 0$ . If  $f$  continues to increase, all individuals move to the highly correlated fraction and the comfort concentrate vanishes.

If the noise of the norm  $\epsilon_k$  is independent of  $\psi$  then the correlation between different  $x_k$  increases monotonically with  $f$ . With an increase of the factor intensity  $f$  the dominant eigenvector of the correlation matrix between  $x_k$  becomes more uniform in the coordinates, which tend asymptotically to  $\pm \frac{1}{\sqrt{m}}$ .

The correlation between systems also increases (just transpose the data matrix), and the coordinates of the dominant eigenvector similarly tend to values  $\frac{1}{\sqrt{n}}$  (which are positive), but this tendency has the character of a “resource exhausting wave” which spreads through the systems following the rule (11).

The observation of Ref. [17] partially supports the uniformity of the eigenvector that corresponds to the largest eigenvalue which “represents the influence of the entire market that is common to all stocks”. Fig. 8d from Ref. [17] shows that the components of this eigenvector are positive and “almost all stocks participate in the largest eigenvector”. Also, in Ref. [11] it was demonstrated that in the periods of drawdowns of the global index (DAX) there appears one strongly dominant eigenvalue for synchronous correlations between 30 companies from DAX. Similar results for 30 British companies are presented in Figs. 8 and 5. In physiology, we also found these “maximum integration” effects for various loads on organisms [5]. When the pressure is lower then, instead of one dominant eigenvector which represents all functional systems of an organism, there appears a group of eigenvectors with relatively high eigenvalues. Each of these vectors has

significant components for attributes of a specific group of functional systems, and the intersection of those groups for different eigenvectors is not large. In addition, the effect of factor “disintegration” because of overload was also observed.

The Selye model describes the first part of the effect (from comfort to stress), but tells us nothing about the other side of crisis near the death.

#### 4.1.3. Mean field realization of the Selye model

In this section we present a simple toy model that is the mean field realization of the Selye model. As a harmful factor for this model we use minus log-return of the FTSE index: the instant value of factor  $f(k)$  at time moment  $k$  is

$$f(t) = -\log(\text{FTSE}(t+1)/\text{FTSE}(t)) \quad (12)$$

This factor could be considered as the mean field produced by the all objects together with some outer sources.

The instant values of stocks log-returns of  $i$ th object  $x_i(k)$  are modeled by the Selye model (11):

$$x_i(t) = -l(f(t) - ar_i)H(f(t) - ar_i) + \epsilon_i(t), \quad (13)$$

where  $H$  is the Heaviside step function.

We compare real data and data for two distributions of resource, Exponential(30) (subscript “exp”) and Uniform(0, 2) (subscript “u”). Random variables  $\epsilon_i(t)$  for various  $i$  and  $t$  are uniformly distributed i.i.d with zero mean and the variance  $\text{var } \epsilon = 0.0035$ . This is the minimum of the average variance of the log-return values for thirty companies. The minimum corresponds to the most “quiet” state of market (in the sense of value of variance) in the time period. We calculated the total variance of 30 companies during the time interval used for analysis (04/07/2007–25/10/2007), found the minimal value of the variance and divided by 30. To compare results for exponential and uniform distributions we use the same realization of noise.

The efficiency coefficient  $a$  is different for different distributions: we calibrate it on such a way that for 75% of objects the value  $ar_i$  is expected to be below  $f$  and 25% are expected to be above  $f$  for the same value of factor  $f$ :  $a_{\text{exp}}/a_u \approx 1.88$ . The ratio of the coefficients  $l_{\text{exp}}/l_u$  should have (approximately) inverse value to keep the expected distances the same for the pairs of objects with  $ar_i < f$ . For qualitative reproduction of the crisis we selected  $a_{\text{exp}} = 0.032$ ,  $a_u = 0.017$ ,  $l_{\text{exp}} = 7.3$ ,  $l_u = 15.5$ .

For each system we calculated the correlation coefficients over the period of 20 days (similar to the analysis made for real data):  $G_{\text{exp}}$ ,  $G_u$ . The right-hand side of the figure represent the dynamics of changes in correlations between objects. Plots in Fig. 11.1a show the number of objects in real data that have more than 1, 2, 4, 8, 16 or 20 values of correlations greater than 0.7, plots in Fig. 11.1b represent the number of companies that have more than 1, 2, 4, 8, 16 or 20 correlations greater than 0.5. Similarly, Figs. 11.2a, b and 11.3a, b represent the model results for the exponential (2) and uniform (3) distributions.

The qualitative character of crisis is reproduced, but the difference from the empirical data is also obvious: the plots for real data also bell-shaped with fluctuations, but they are wider than the model curves and fluctuations do not go to zero outside the crisis period in reality. The simplest improvement of the situation may be achieved by introduction of correlated noise and fitting. In the simplest Selye model we assume zero correlations in the comfort zone but in reality the correlations do not decrease to zero.

Amplitude of noise differs for different companies and we can take its distribution from empirical data. Coefficient  $l$  in the basic Selye model (10) also depends on the company but in the toy model we take it constant.

One problem exists for all these improvements: they introduce too many parameters for fitting. Of course, more degrees of freedom available for fitting give more flexibility in quantitative approximation of the empirical data. The simplest toy model has two parameters only.

Another way to improvement is the selection of a better mean field factor. Now we make just a first choice and selected the negative log-return of the FTSE index as a mean-field harmful factor. The serious modification of model could take into account the pressure of several factors too.

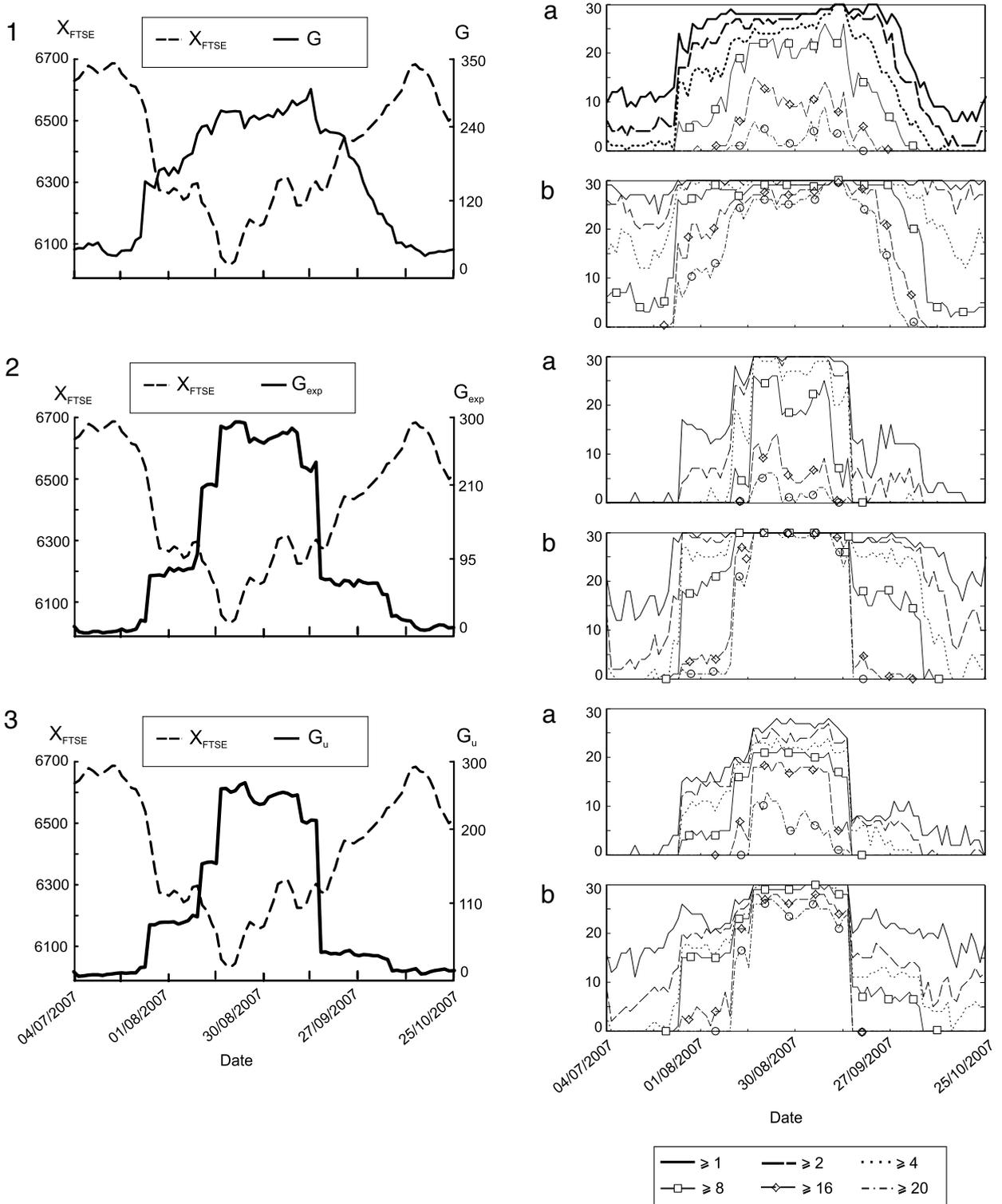
#### 4.1.4. How to merge factors?

Usually, there are many factors. Formally, for  $q$  factors one can generalize the one-factor tension-driven model (10) in the form.

$$x_k = x_k(\psi_1, \psi_2, \dots, \psi_q) + \epsilon_k. \quad (14)$$

In this equation, the compensated values of factors,  $\psi_i = f_i - a_i r_i$ , are used and  $\sum_{i=1}^q r_i \leq R$ .

Two questions appear immediately: (i) how to find the distribution of resource, assigned for neutralization of different factors, and (ii) how to represent the functions  $x_k(\psi_1, \dots, \psi_q)$ . Usually, in factor analysis and in physics both, we start from the assumption of linearity (“in the first approximation”), but this approximation does not work here properly. In the simplest reasonable approximation, max–min operations appear instead of linear operations. This sounds very modern [55] and even a bit extravagant, but it was discovered many years ago by Justus von Liebig (1840). His “law of the minimum” states that growth is controlled by the scarcest resource (limiting factor) [56]. This concept was originally applied to plant or crop growth. Many times it was criticized, rejected, and then returned and demonstrated quantitative agreement with experiments [56–58]. Liebig’s Law of the minimum was extended to more a general conception of factors, not only for



**Fig. 11.** The dynamics of indicators of correlation matrices for (1) real data, (2) system with exponentially distributed resources, (3) system with uniformly distributed resources. The left-hand part represents the general dynamics of  $G$ ,  $G_{exp}$ ,  $G_u$  in comparison to the dynamics of FTSE over the time period 04/07/2007–25/10/2007. The right-hand part shows the dynamics of changes in correlations between objects over the interval: (a) number of objects that have more than 1, 2, 4, 8, 16 or 20 values of correlations greater than 0.7, (b) number of objects that have more than 1, 2, 4, 8, 16 or 20 values of correlations greater than 0.5.

elementary physical description of available chemical substances and energy. Any environmental factor essential for life that is below the critical minimum, or that exceeds the maximum tolerable level could be considered as a limiting one.

The biological generalizations of Liebig’s Law were supported by the biochemical idea of limiting reaction steps (the modern theory of limiting steps and dominant systems for multiscale reaction networks is presented in the recent review [59]). Some of the generalizations went quite far from agriculture and ecology. The law of the minimum was applied to economics [60] and to education, for example [61].

According to Liebig’s Law, the tension-driven model is

$$x_k = \mu_k + l_k \max_{1 \leq i \leq q} \{\psi_i\} + \epsilon_k. \tag{15}$$

This model seems to be linear, but its nonlinearity is hidden in dependence of  $\psi_i$  on the distribution of factors and the amount of the resource available.

#### 4.1.5. Optimality and fitness

Adaptation optimizes the state of the system for a given amount of the resource available. It may be difficult to find the objective function that is hidden behind the adaptation process. Nevertheless, even an assumption about the existence of an objective function and about its general properties helps in the analysis of the adaptation process. Assume that adaptation should maximize an objective function  $W$  which depends on the compensated values of factors,  $\psi_i = f_i - a_i r_i$  for the given amount of available resource:

$$\begin{cases} W(f_1 - a_1 r_1, f_2 - a_2 r_2, \dots, f_q - a_q r_q) \rightarrow \max; \\ r_i \geq 0, \quad f_i - a_i r_i \geq 0, \quad \sum_{i=1}^q r_i \leq R. \end{cases} \tag{16}$$

The only question is: why can we be sure that adaptation follows any optimality principle? Existence of optimality is proven for microevolution processes and ecological succession. The mathematical backgrounds for the notion of “natural selection” in these situations are well established after works of Haldane (1932) [29] and Gause (1934) [30]. Now this direction with various concepts of *fitness* (or “generalized fitness”) optimization is elaborated in many details (see, for example, review papers [31–33]).

The foundation of optimization is not so clear for such processes as modifications of phenotype, and for adaptation in various time scales. The idea of *genocopy–phenocopy interchangeability* was formulated long ago by biologists to explain many experimental effects: the phenotype modifications simulate the optimal genotype [34, p. 117]. The idea of convergence of genetic and environmental effects was supported by analysis of genome regulation [35] (the principle of concentration–affinity equivalence). The phenotype modifications produce the same change, as evolution of genotype does, but faster and in a smaller range of conditions (the proper evolution can go further, but slower). It is natural to assume that adaptation in different time scales also follows the same direction, as evolution and phenotype modifications, but faster and for smaller changes. This hypothesis could be supported by many biological data and plausible reasoning. For social and economical systems the idea of optimization of individual behavior seems very natural. The selection arguments may be also valid for such systems.

It seems productive to accept the idea of optimality, and to use it, as far as this will not contradict the data.

#### 4.2. Law of the minimum paradox

Liebig used the image of a barrel – now called Liebig’s barrel – to explain his law. Just as the capacity of a barrel with staves of unequal length is limited by the shortest stave, so a plant’s growth is limited by the nutrient in shortest supply. An adaptation system acts as a cooper and repairs the shortest stave to improve the barrel capacity. Indeed, in well-adapted systems the limiting factor should be compensated as far as this is possible. It seems obvious because of the very natural idea of optimality, but arguments of this type in biology should be considered with care.

Assume that adaptation should maximize a objective function  $W$  (16) which satisfies Liebig’s Law:

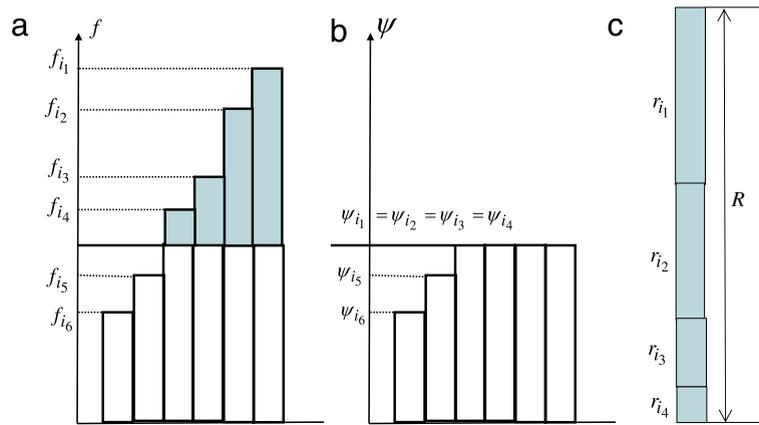
$$W = W \left( \max_{1 \leq i \leq q} \{f_i - a_i r_i\} \right); \quad \frac{\partial W(x)}{\partial x} \leq 0 \tag{17}$$

under conditions  $r_i \geq 0, f_i - a_i r_i \geq 0, \sum_{i=1}^q r_i \leq R$ . (Let us recall that  $f_i \geq 0$  for all  $i$ .)

Description of the maximizers of  $W$  gives the following theorem (the proof is a straightforward consequence of Liebig’s Law and monotonicity of  $W$ ).

**Theorem 1.** For any objective function  $W$  that satisfies conditions (17) the optimizers  $r_i$  are defined by the following algorithm.

- (1) Order intensities of factors:  $f_{i_1} \geq f_{i_2} \geq \dots \geq f_{i_q}$ .
- (2) Calculate differences  $\Delta_j = f_{i_j} - f_{i_{j+1}}$  (take formally  $\Delta_0 = \Delta_{q+1} = 0$ ).



**Fig. 12.** Optimal distribution of resource for neutralization of factors under Liebig's Law. (a) histogram of factors intensity (the compensated parts of factors are highlighted,  $k = 3$ ), (b) distribution of tensions  $\psi_i$  after adaptation becomes more uniform, (c) the sum of distributed resources. For simplicity of the picture, we take here all  $a_i = 1$ .

(3) Find such  $k$  ( $0 \leq k \leq q$ ) that

$$\sum_{j=1}^k \left( \sum_{p=1}^j \frac{1}{a_p} \right) \Delta_j \leq R \leq \sum_{j=1}^{k+1} \left( \sum_{p=1}^j \frac{1}{a_p} \right) \Delta_j.$$

For  $R < \Delta_1$  we put  $k = 0$ .

(4) If  $k < q$  then the optimal amount of resource  $r_{j_l}$  is

$$r_{j_l} = \begin{cases} \frac{\Delta_l}{a_l} + \frac{1}{a_l \sum_{p=1}^k \frac{1}{a_p}} \left( R - \sum_{j=1}^k \left( \sum_{p=1}^j \frac{1}{a_p} \right) \Delta_j \right), & \text{if } l \leq k + 1; \\ 0, & \text{if } l > k + 1. \end{cases} \quad (18)$$

If  $k = q$  then  $r_i = f_i$  for all  $i$ .  $\square$

Here  $a_p$  stands for  $a_{jp}$ . This optimization is illustrated in Fig. 12.

Hence, if the system satisfies the law of the minimum then the adaptation process makes the tension produced by different factors  $\psi_i = f_i - ar_i$  (Fig. 12) more uniform. Thus adaptation decreases the effect of the limiting factor and hides manifestations of Liebig's Law.

Under the assumption of optimality (16) the law of the minimum paradox becomes a theorem: if Liebig's Law is true then microevolution, ecological succession, phenotype modifications and adaptation decrease the role of the limiting factors and bring the tension produced by different factors together.

The cooper starts to repair Liebig's barrel from the shortest stave and after reparation the staves are more uniform, than they were before. This cooper may be microevolution, ecological succession, phenotype modifications, or adaptation. For the ecological succession this effect (Liebig's Law leads to its violation by succession) was described in Ref. [62]. For adaptation (and in general settings too) it was demonstrated in Ref. [2].

The law of the minimum together with the idea of optimality (even without an explicit form of the objective function) gives us answers to both question: (i) we now know the optimal distribution of the resource (18), assigned for neutralization of different factors, and (ii) we can choose the function  $x_k(\psi_1, \dots, \psi_q)$  from various model forms, the simplest of them gives the tension-driven models (15).

#### 4.3. Law of the minimum inverse paradox

The simplest formal example of "anti-Liebig's" organization of interaction between factors gives us the following dependence of fitness from two factors:  $W = -f_1 f_2$ : each of the factors is neutral in the absence of another factor, but together they are harmful. This is an example of *synergy*: the whole is greater than the sum of its parts. (For our selection of axes direction, "greater" means "more harm".) Let us give the formal definition of the synergistic system of factors for the given fitness function  $W$ .

**Definition.** The system of factors  $F_1, \dots, F_q$  is synergistic, if for any two different vectors of their admissible values  $\mathbf{f} = (f_1, \dots, f_q)$  and  $\mathbf{g} = (g_1, \dots, g_q)$  ( $\mathbf{f} \neq \mathbf{g}$ ) the value of fitness at the average point  $(\mathbf{f} + \mathbf{g})/2$  is less, than at the best

of points  $\mathbf{f}, \mathbf{g}$ :

$$W\left(\frac{\mathbf{f} + \mathbf{g}}{2}\right) < \max\{W(\mathbf{f}), W(\mathbf{g})\}. \tag{19}$$

Liebig’s systems of factors violate the synergy inequality (19): if at points  $\mathbf{f}, \mathbf{g}$  with the same values of fitness  $W(\mathbf{f}) = W(\mathbf{g})$  different factors are limiting, then at the average point the value of both these factors are smaller, and the harm of the limiting factor at that point is less, than at both points  $\mathbf{f}, \mathbf{g}$ , i.e. the fitness at the average point is larger.

The fitness function  $W$  for synergistic systems has a property that makes the solution of optimization problems much simpler. This proposition follows from the definition of convexity and standard facts about convex sets (see, for example, [63]).

**Proposition 1.** *The synergy inequality (19) holds if and only if all the sublevel sets  $\{\mathbf{f} | W(\mathbf{f}) \leq \alpha\}$  are strictly convex.  $\square$*

(The fitness itself may be a non-convex function.)

This proposition immediately implies that the synergy inequality is invariant with respect to increasing monotonic transformations of  $W$ . This invariance with respect to nonlinear change of scale is very important, because usually we don’t know the values of function  $W$ .

**Proposition 2.** *If the synergy inequality (19) holds for a function  $W$ , then it holds for a function  $W_\theta = \theta(W)$ , where  $\theta(x)$  is an arbitrary strictly monotonic function of one variable.  $\square$*

Already this property allows us to study the problem about optimal distribution of the adaptation resource without further knowledge about the fitness function.

Assume that adaptation should maximize an objective function  $W(f_1 - r_1, \dots, f_q - r_q)$  (16) which satisfies the synergy inequality (19) under conditions  $r_i \geq 0, f_i - a_i r_i \geq 0, \sum_{i=1}^q r_i \leq R$ . (Let us remind that  $f_i \geq 0$  for all  $i$ .) Following our previous convention about axes directions all factors are harmful and  $W$  is monotonically decreasing function

$$\frac{\partial W(f_1, \dots, f_q)}{\partial f_i} < 0.$$

We need also a technical assumption that  $W$  is defined on a convex set in  $\mathbb{R}_+^q$  and if it is defined for a nonnegative point  $\mathbf{f}$ , then it is also defined at any nonnegative point  $\mathbf{g} \leq \mathbf{f}$  (this inequality means that  $g_i \leq f_i$  for all  $i = 1, \dots, q$ ).

The set of possible maximizers is finite. For every group of factors  $F_{i_1}, \dots, F_{i_{j+1}}, (1 \leq j + 1 < q)$  with the property

$$\sum_{k=1}^j \frac{f_{i_k}}{a_{i_k}} < R \leq \sum_{k=1}^{j+1} \frac{f_{i_k}}{a_{i_k}} \tag{20}$$

we find a distribution of resource  $\mathbf{r}_{\{i_1, \dots, i_{j+1}\}} = (r_{i_1}, \dots, r_{i_{j+1}})$ :

$$r_{i_k} = \frac{f_{i_k}}{a_{i_k}} \ (k = 1, \dots, j), \quad r_{i_{j+1}} = R - \sum_{k=1}^j \frac{f_{i_k}}{a_{i_k}}, \quad r_i = 0 \ \text{for } i \notin \{i_1, \dots, i_{j+1}\}. \tag{21}$$

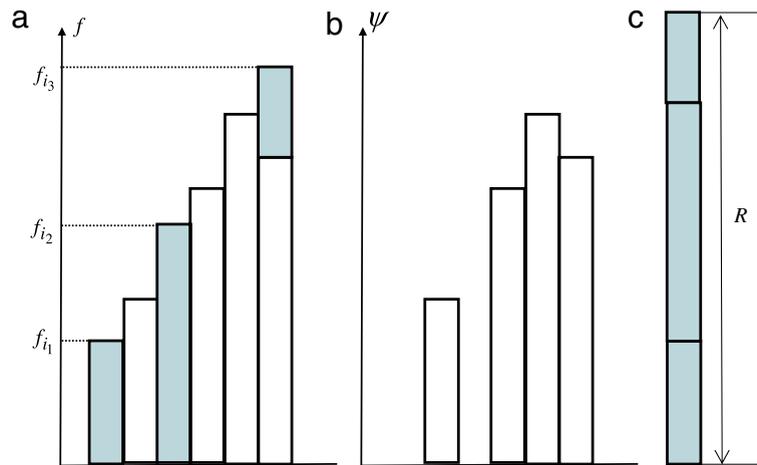
For  $j = 0$ , Eq. (20) gives  $0 < R \leq f_{i_1}/a_{i_1}$  and there exists only one nonzero component in the distribution (21),  $r_{i_1} = R$ .

We get the following theorem as an application of standard results about extreme points of convex sets [63].

**Theorem 2.** *Any maximizer for  $W(f_1 - r_1, \dots, f_q - r_q)$  under given conditions has the form  $\mathbf{r}_{\{i_1, \dots, i_{j+1}\}}$  (21).  $\square$*

If the initial distribution of factors intensities,  $\mathbf{f} = (f_1, \dots, f_q)$ , is almost uniform and all factors are significant then, after adaptation, the distribution of effective tensions,  $\boldsymbol{\psi} = (\psi_1, \dots, \psi_q)$  ( $\psi_i = f_i - a_i r_i$ ), is less uniform. Following Theorem 2, some of the factors may be completely neutralized and one additional factor may be neutralized partially. This situation is opposite to adaptation due to Liebig’s system of factors, where the amount of significant factors increases and the distribution of tensions becomes more uniform because of adaptation. For Liebig’s system, adaptation transforms a low-dimensional picture (one limiting factor) into a high-dimensional one, and we expect the well-adapted systems have less correlations than in stress. For synergistic systems, adaptation transforms a high-dimensional picture into a low-dimensional one (less factors), and our expectations are inverse: we expect the well-adapted systems have more correlations than in stress (this situation is illustrated in Fig. 13; compare to Fig. 12). We call this property of adaptation to synergistic system of factors the *law of the minimum inverse paradox*.

Fitness by itself is a theoretical construction based on the average reproduction coefficient (instant fitness). It is impossible to measure this quantity in time intervals that are much shorter than life length. Hence, to understand which system of factors we deal with, Liebig’s or a synergistic one, we have to compare the theoretical consequences of their properties. First of all, we can measure the results of adaptation, and use properties of the optimal adaptation in ensembles of systems for analysis (Figs. 12 and 13).



**Fig. 13.** Typical optimal distribution of resource for neutralization of synergistic factors. (a) Factors intensity (the compensated parts of factors are highlighted,  $j = 2$ ), (b) distribution of tensions  $\psi_i$  after adaptation becomes less uniform (compare to Fig. 12), (c) the sum of distributed resources. For simplicity of the picture, we take here all  $a_i = 1$ .

There is some evidence about the existence of synergistic systems of factors. For example, the postsurgical rehabilitation of people suffering lung cancer of the III and IV clinical groups was studied [1]. Dynamics of variance and correlations for them have directions which are unusual for Liebig's systems: increase of the correlation corresponds to decrease of the variance. Moreover, analysis of the maxima and minima of correlations and mortality demonstrates that in this case an increase of correlations corresponds to decrease of stress. Hence, in Ref. [1] the hypothesis was suggested that in this case some factors superlinearly increase the harmfulness of other factors, and this is an example of a synergistic system of factors. Thus, the law of the minimum inverse paradox may give us a clue to the effect (Fig. 1) near the fatal outcomes.

## 5. Discussion

### 5.1. Dynamics of the correlations in crisis

We study a universal effect in ensembles of similar systems under load of similar factors: in crisis, typically, correlation increases, and, at the same time, variance (and volatility) increases too. This effect is demonstrated for humans, mice, trees, grassy plants, and financial time series. It is represented as the left transition in Fig. 1, the transition from comfort to crisis. Already a system of simple models of adaptation to one factor (we call it the *Selye model*) gives a qualitative explanation of the effect.

For interaction of several factors two basic types of organization are considered: Liebig's systems and synergistic systems of factors. The adaptation process (as well as phenomodification, ecological succession, or microevolution) acts differently onto these systems of factors and makes Liebig's systems more uniform (instead of systems with limiting factor) and synergistic systems less uniform. These theorems give us two paradoxes which explain differences observed between artificial (less adapted) systems and natural (well-adapted) systems.

Empirically, we expect the appearance of synergistic systems in extremely difficult conditions, when factors appear that superlinearly amplify the harm from other factors. This means that after the crisis achieves its bottom, it can develop into two directions: recovering (both correlations and variance decrease) or fatal catastrophe (correlations decrease, but variance not). The transition to fatal outcome is represented as the right transition in Fig. 1. Some clinical data support these expectations.

### 5.2. Correlations between the thirty largest FTSE companies

The case study of the thirty largest companies from British stock market for the period 2006–2008 supports the hypothesis about increasing of the correlations in crisis. It is also demonstrated that the correlation in time (between daily data) also has diagnostic power (as well as the correlation between companies has) and connections between days (Figs. 9 and 10) may clearly indicate and, sometimes, predict the chronology of the crisis. This approach (use of two time moments instead of the time window) allows to overcome a smearing effect caused by usage of time windows (about this problem see Refs. [42,43]).

The principal component analysis demonstrates that the largest eigenvalues of the correlation matrices increase in crisis and under environmental pressure (before the inverse effect "on the other side of crisis" appears). Different methods for selection of significant principal components, Kaiser's rule, random matrix approach and the broken stick model, give similar

results in a case study. Kaiser's rule gives more principal components than two other methods and the higher sensitivity of the indicator  $\text{Dim}_K$  causes some difficulties in interpretation. The random matrix estimates select too small amount of components, and the indicator  $\text{Dim}_M P$  seems not sensitive enough. In our case study the best balance between sensitivity and stability gives the dimension, estimated by the broken stick model  $\text{Dim}_{BS}$ .

### 5.3. Choice of coordinates and the problem of invariance

All indicators of the level of correlations are non-invariant with respect to transformations of coordinates. For example, rotation to the principal axis annuls all the correlations. Dynamics of variance also depends on nonlinear transformations of scales. Dimensionless variance of logarithms (or "relative variance") often demonstrates more stable behavior especially when changes of mean values are large.

The observed effect depends on the choice of attributes. Nevertheless, many researchers observed it without a special choice of coordinate system. What does it mean? We can propose a hypothesis: the effect may be so strong that it is almost improbable to select a coordinate system where it vanishes. For example, if one accepts the Selye model (10), (11) then observability of the effect means that for typical nonzero values of  $\psi$  in crisis

$$l_k^2 \psi^2 > \text{var}(\epsilon_k) \quad (22)$$

for more than one value of  $k$ , where  $\text{var}$  stands for variance of the noise component (this is sufficient for increase of the correlations). If

$$\psi^2 \sum_k l_k^2 \gg \sum_k \text{var}(\epsilon_k)$$

and the set of allowable transformations of coordinates is bounded (together with the set of inverse transformations), then the probability to select randomly a coordinate system which violates condition (22) is small (for reasonable definitions of this probability and of the relation  $\gg$ ). On another hand, the choice of attributes is never random, and one can look for the reason of so wide observability of the effect in our (human) ways to construct the attribute systems.

### 5.4. Two programs for further research

First of all, the system of simple models of adaptation should be fitted to various data, both economical and biophysical. Classical econometrics [64] already deals with hidden factors, now we have just to fit a special nonlinear model of adaptation to these factors.

Another possible direction is the development of dynamical models of adaptation. In the present form the model of an adaptation describes a single action, distribution of adaptation resource. We avoid any kinetic modeling. Nevertheless, adaptation is a process in time. We have to create a system of models with a minimal number of parameters.

Models of individual adaptation could explain effects caused by external factors or individual internal factors. They can be also used with the mean-field models when the interaction between systems is presented as an additional factor. The models of interaction need additional hypotheses and data. In this paper, we do not discuss such models, but in principle they may be necessary, because crisis may be caused not by purely external factors but by combination of external factors, individual internal dynamics and interaction between systems.

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