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IEEE World Congress on Computational Intelligence
Virtual Conference – July 19-24, 2020

Augmented AI: correctors of errors and social networks of AI

Alexander N. Gorban

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Training Neural Networks, USSR-USA JV ParaGraph, **1990**

Neural Networks on PC, Nauka, **1996**

Neuoinformatics, Nauka, **1998**

1978-2003

Russian Academy of Sciences, Siberian Branch

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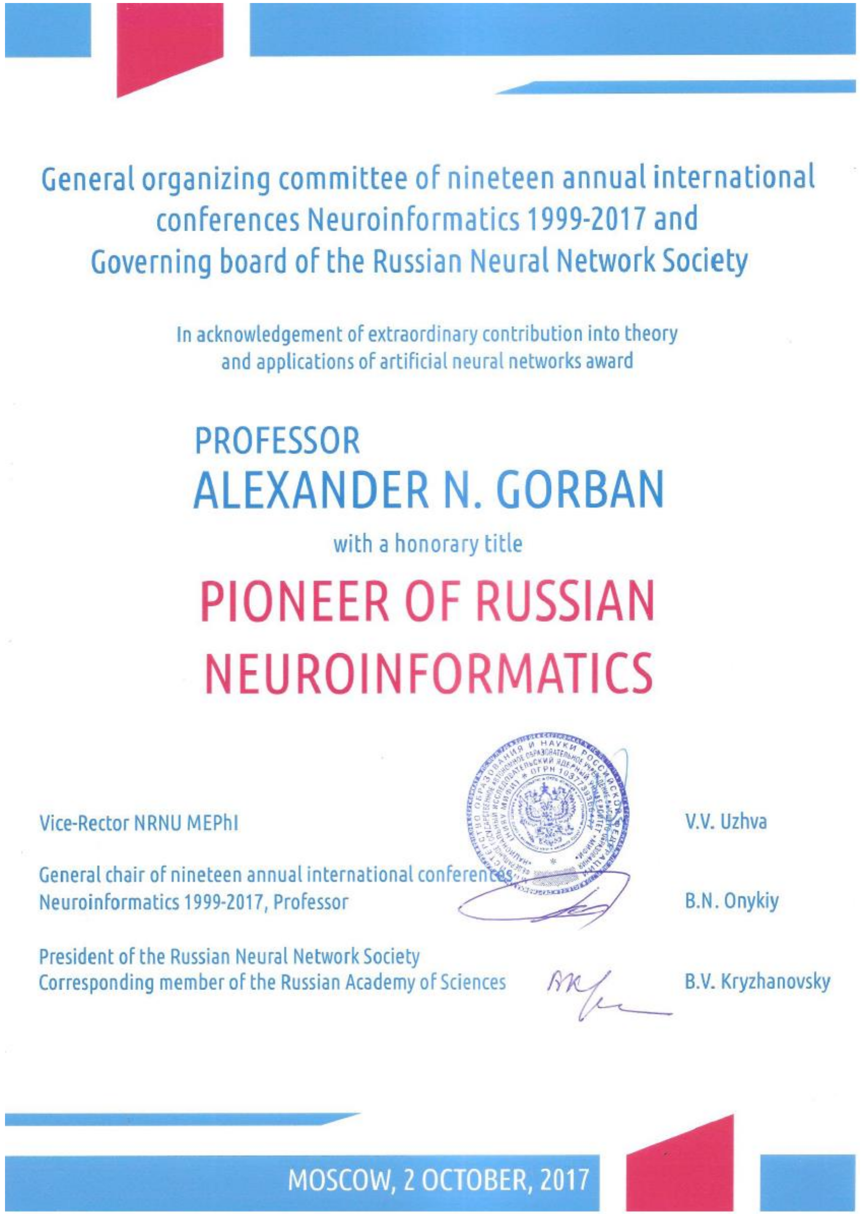
General organizing committee of nineteen annual international conferences Neuroinformatics 1999-2017 and Governing board of the Russian Neural Network Society

In acknowledgement of extraordinary contribution into theory and applications of artificial neural networks award

PROFESSOR
ALEXANDER N. GORBAN
with a honorary title
PIONEER OF RUSSIAN NEUROINFORMATICS

Vice-Rector NRNU MEPhI — V.V. Uzhva

General chair of nineteen annual international conferences Neuroinformatics 1999-2017, Professor — B.N. Onykiy

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MOSCOW, 2 OCTOBER, 2017

1978-2003
Russian Academy of Sciences, Siberian Branch

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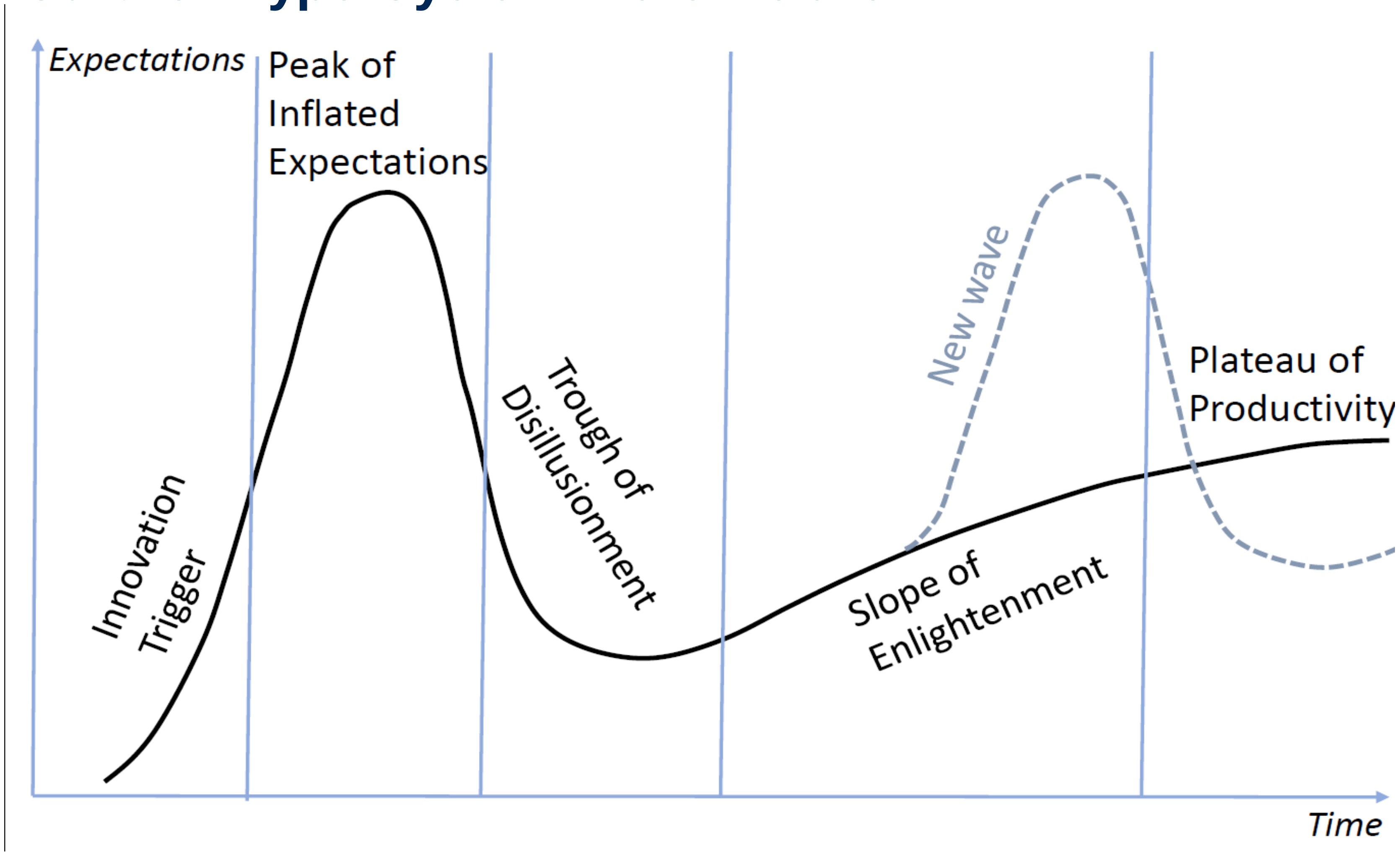
A problem about the problem

A problem about the problem

Gartner Hype Cycle: where we are?



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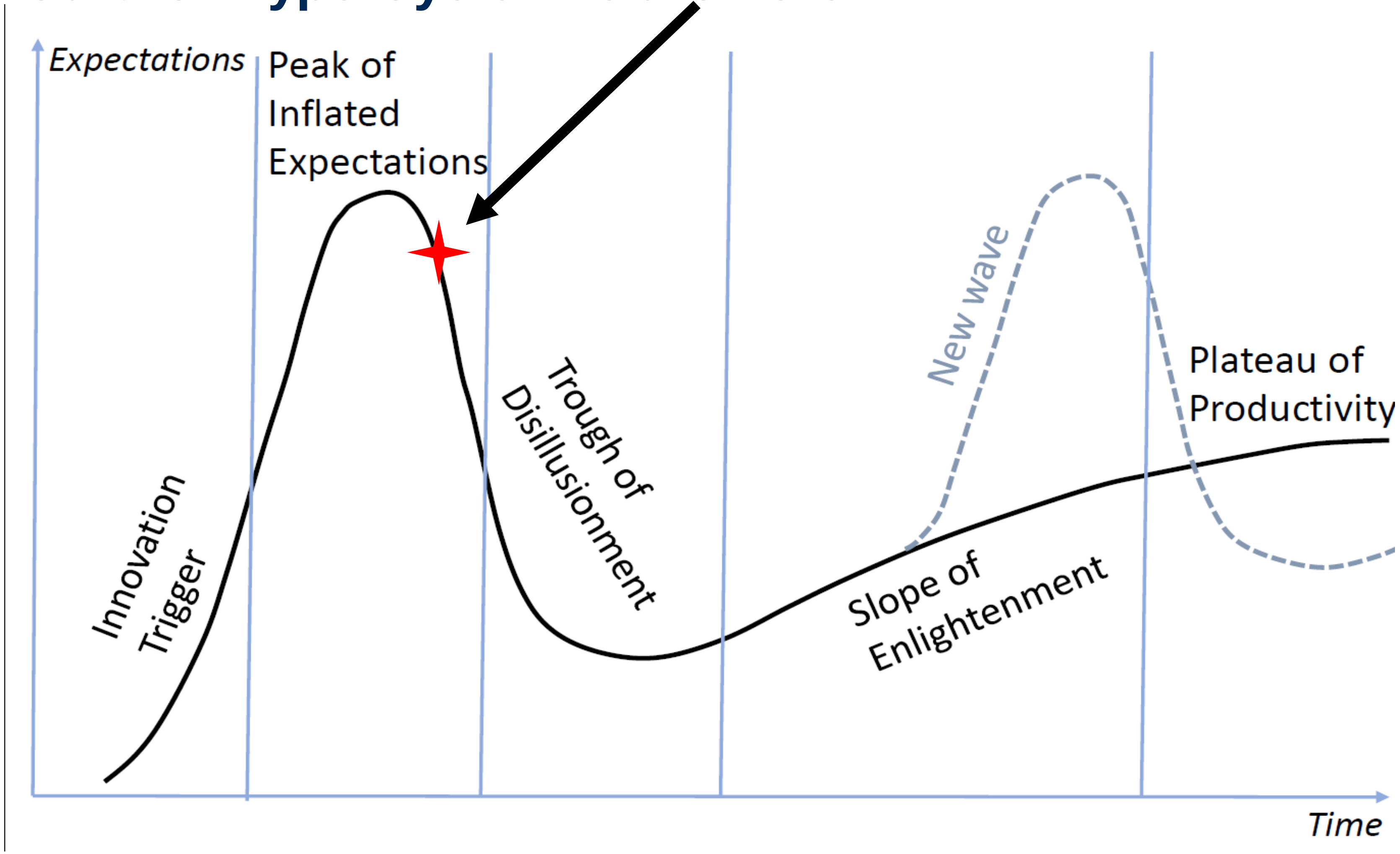

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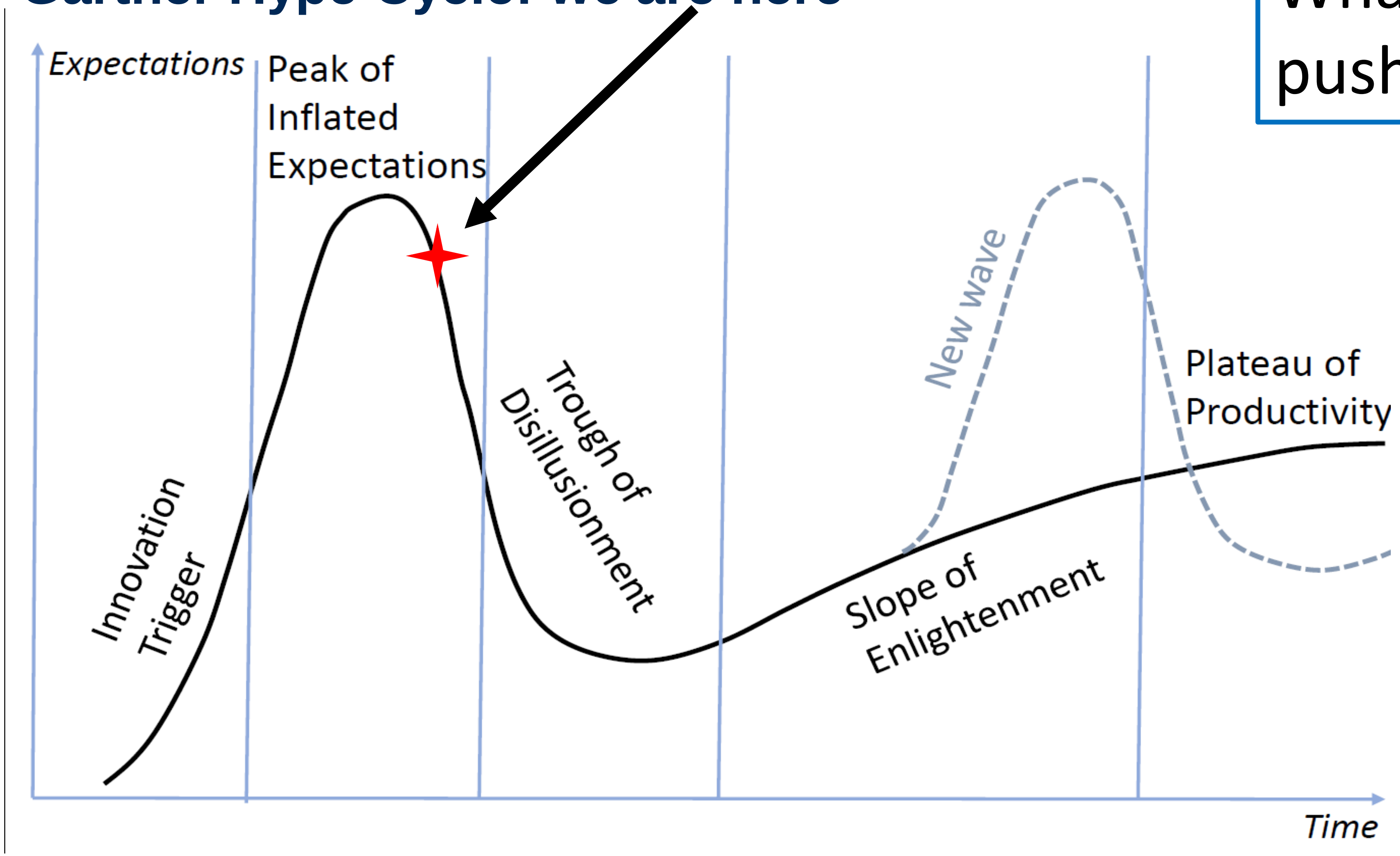

What problems will push us down the hill?

A problem about the problem

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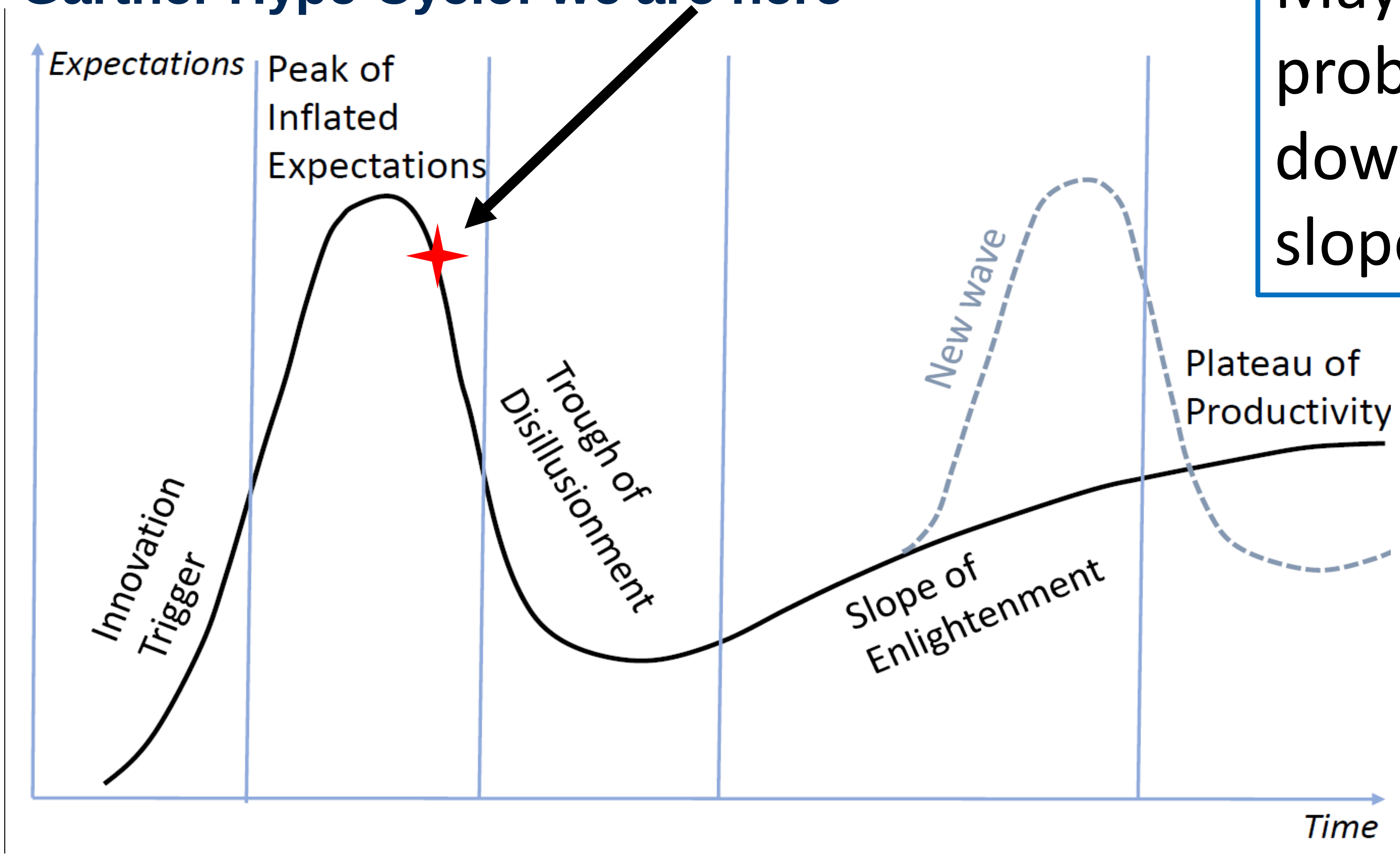

Maybe this is the same problem that pushed us down the previous slopes, decades ago?

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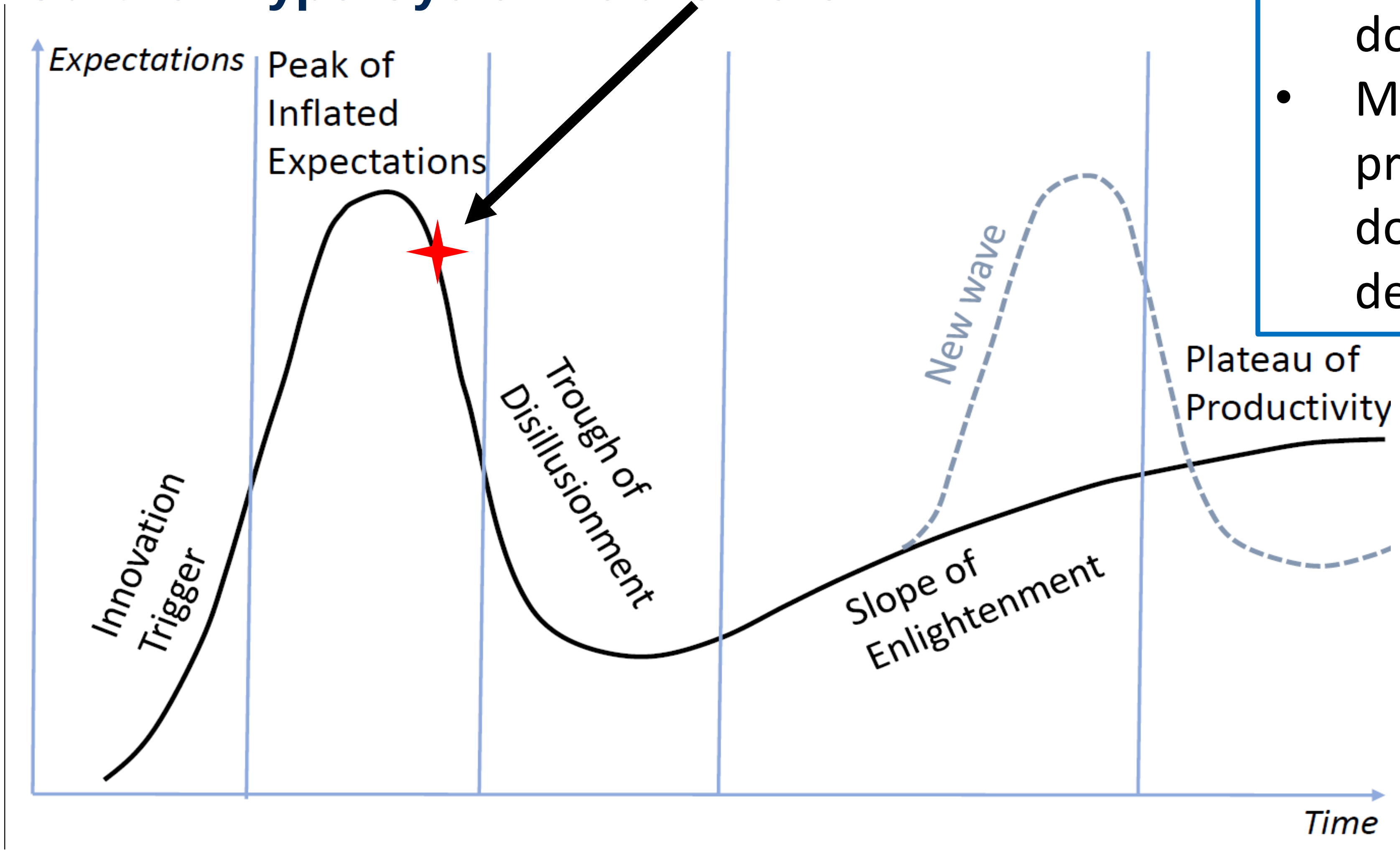

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I try to answer these questions about problems

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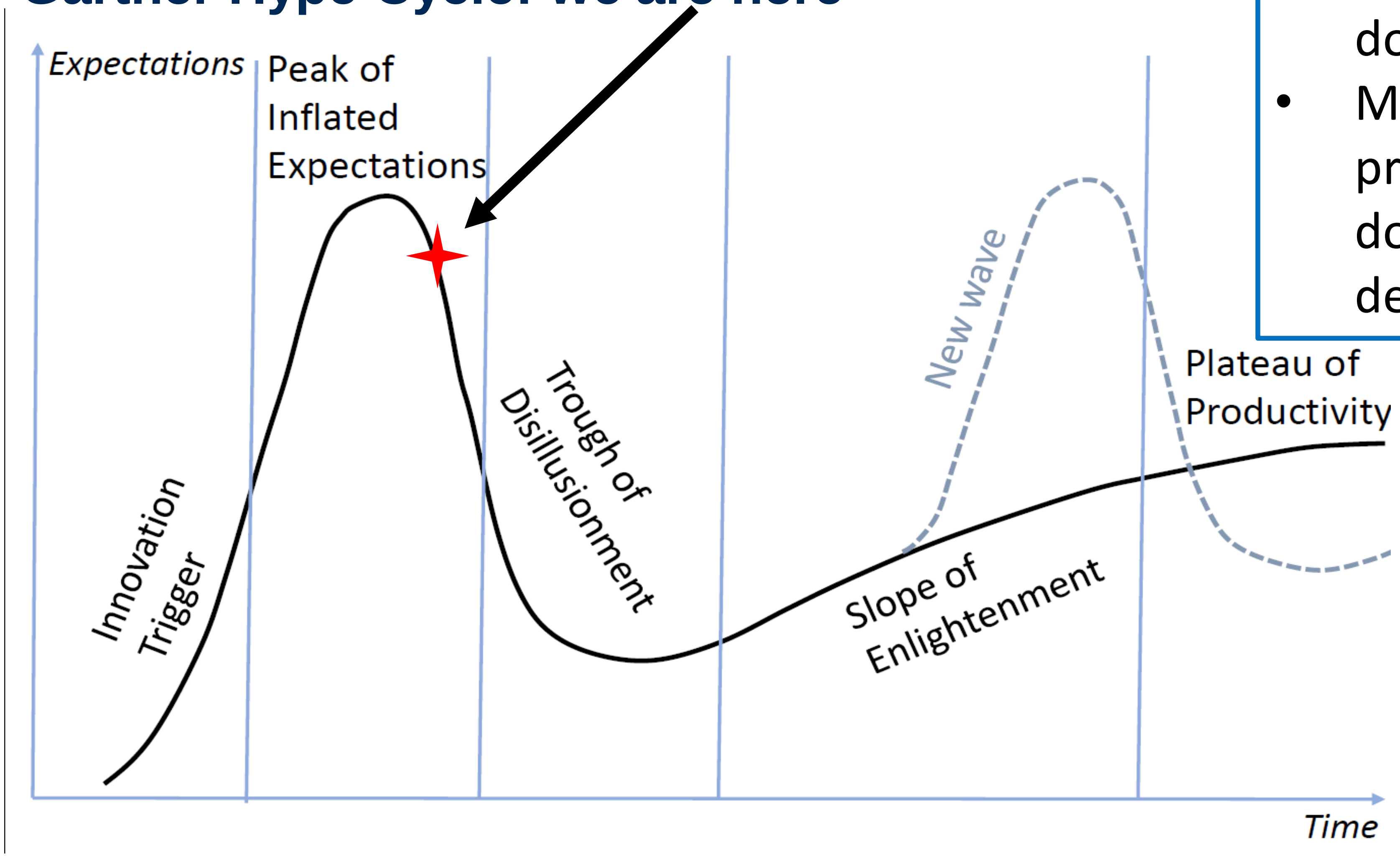

- What problems will push us down the hill?
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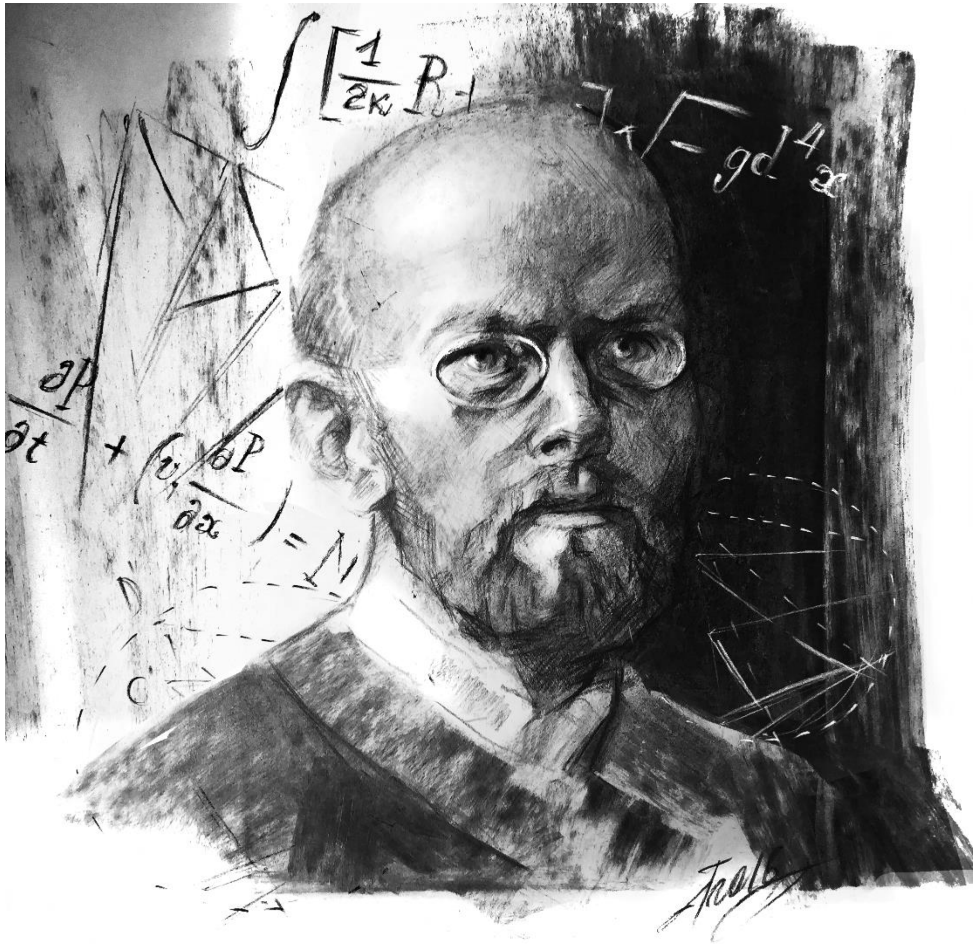
and give some ideas on how to solve these problems.

World congress is a proper place to discuss problems

In Mathematics, this is a long tradition.

David Hilbert, Portrait by Anna Gorban

World congress is a proper place to discuss problems



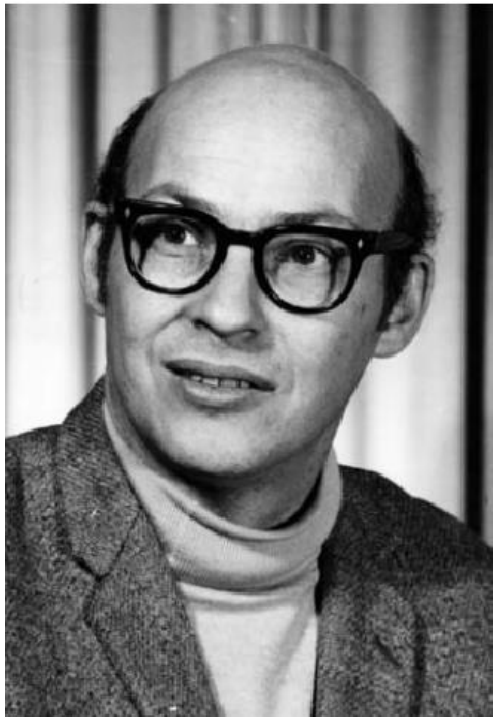
David Hilbert in 1900, Portrait by Anna Gorban

In Mathematics, this is a long tradition.

A famous example:

- In 1900, Hilbert presented his problems at the International Congress of Mathematicians.
- These problems stimulated the development of mathematics and focused the efforts of many mathematicians

Marvin Minsky's Problems for AI

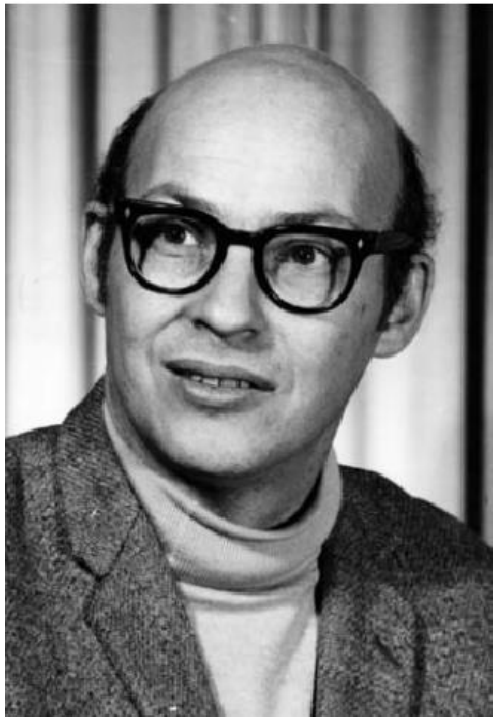


Marvin Minsky,
Semantic Scholar
courtesy MIT Museum,

"It is convenient to divide the problems into five main areas:

- Search,
- Pattern-Recognition,
- Learning,
- Planning,
- and Induction… .

Minsky, M. (1961). Steps toward artificial intelligence. *Proceedings of the IRE*, *49*(1), 8-30.



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- Search,
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- Minsky's problems focused research on the most important technical issues.
- We are currently seeing great success in these areas.

Minsky, M. (1961). Steps toward artificial intelligence. *Proceedings of the IRE*, *49*(1), 8-30.

Two pebbles in a shoe

- A mountain of success of Neural AI inspired us!

Two pebbles in a shoe

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- However, two pebbles in our shoes can slow down this fantastically successful movement:

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AI makes unexpected mistakes, and will make them in the future;

Two pebbles in a shoe

- A mountain of success of Neural AI inspired us!
- However, two pebbles in our shoes can slow down this fantastically successful movement:

AI makes unexpected mistakes, and will make them in the future;

Decisions of Neural AI are not transparent and, therefore, cannot be explained logically.

The main problem for the widespread use of AI around the world are unexpected errors

- The mistakes can be dangerous;
- Usually, it remains unclear, who is responsible for them;
- To avoid a recurrence of a detected error, a quick, non-iterative system fix is required;
- Correction of errors should not damage existing skills;
- The real world is not a good i.i.d. sample,

  -the types of errors are very numerous and often unpredictable,

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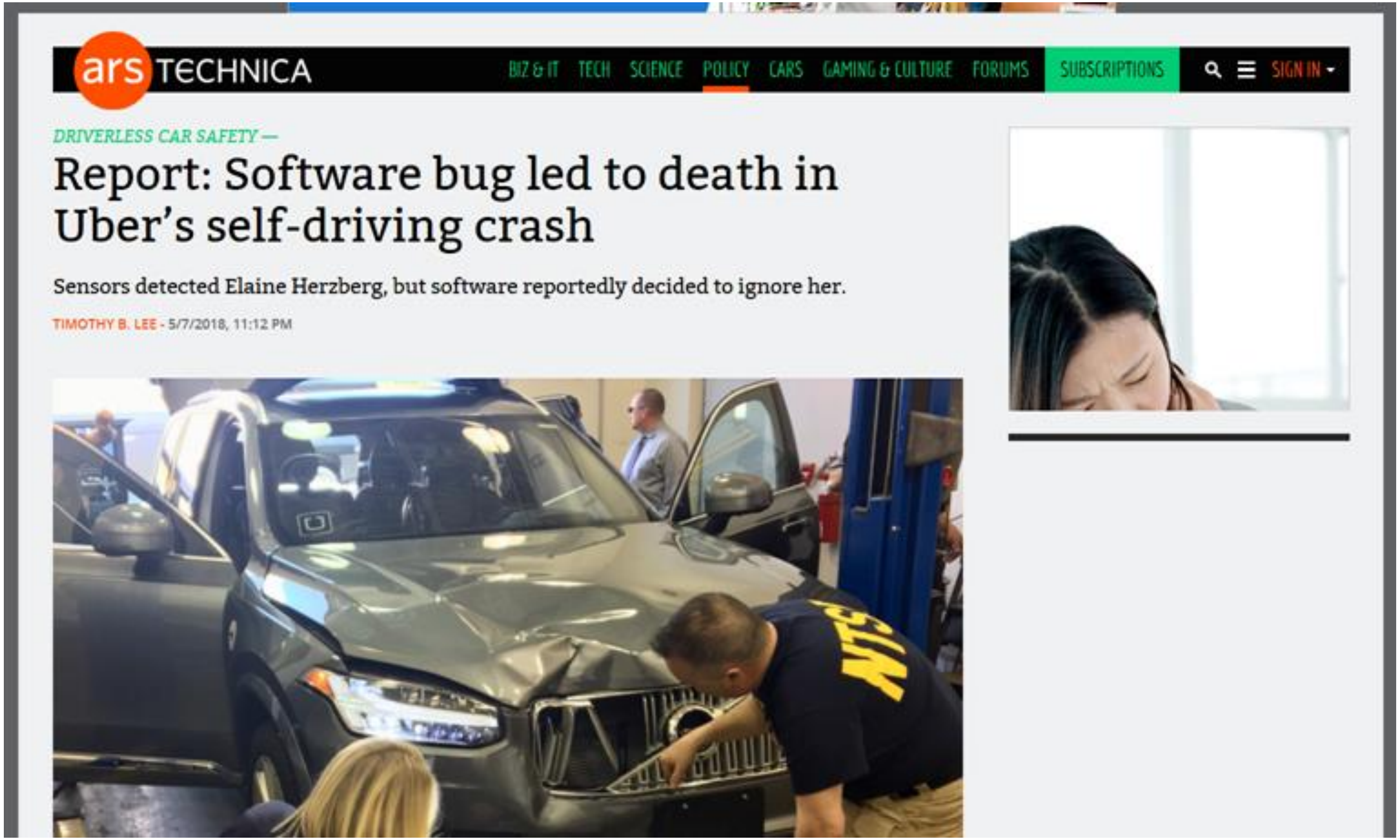

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DRIVERLESS CAR SAFETY —

Report: Software bug led to death in Uber's self-driving crash

Sensors detected Elaine Herzberg, but software reportedly decided to ignore her.

TIMOTHY B. LEE - 5/7/2018, 11:12 PM

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A Tesla electric car crashed into a highway barrier in Mountain View, California, on March 23, 2018. Investigators confirmed that Autopilot was partially to blame.

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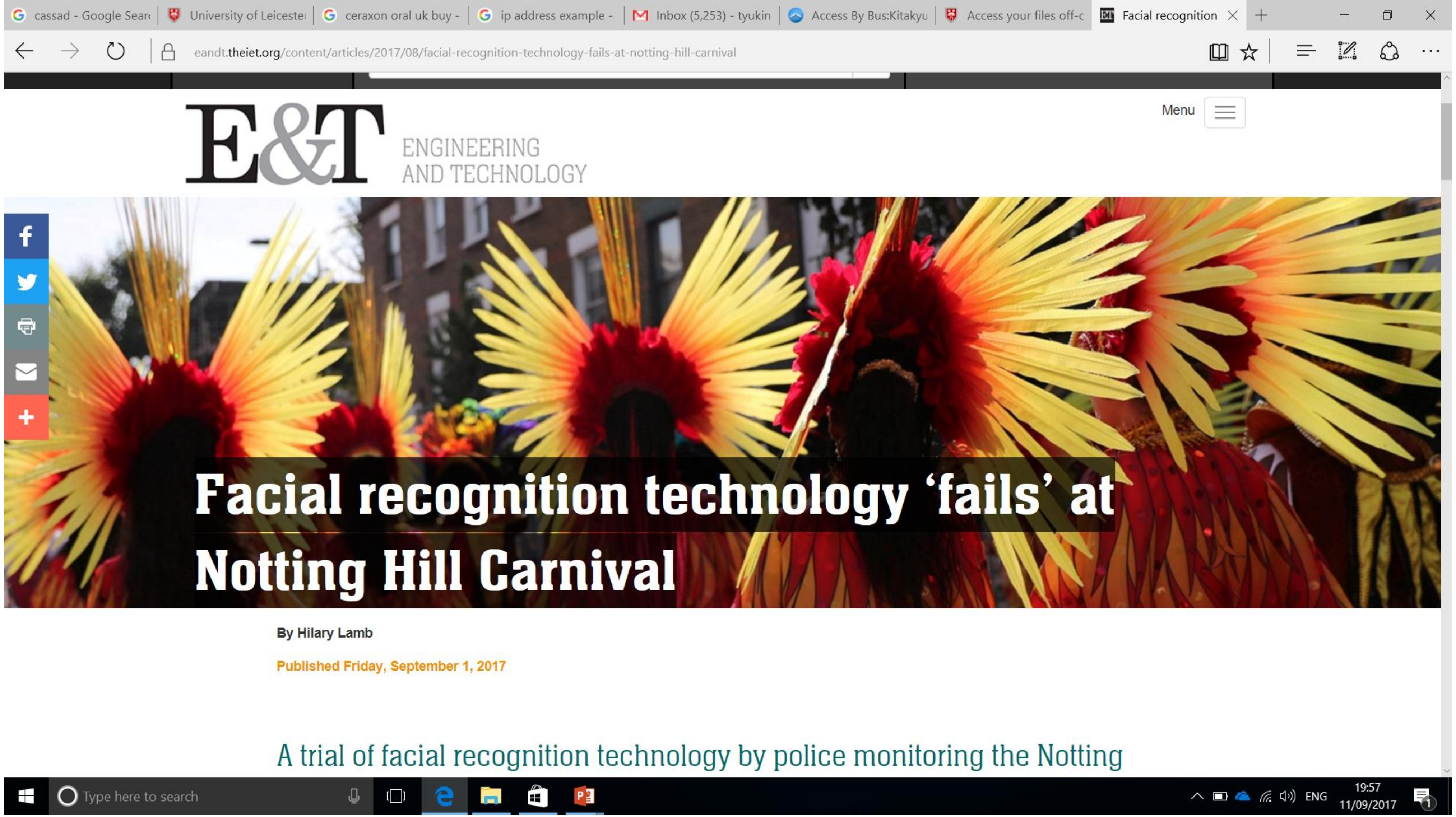

eandt.theiet.org/content/articles/2017/08/facial-recognition-technology-fails-at-notting-hill-carnival

E&T ENGINEERING AND TECHNOLOGY

Facial recognition technology 'fails' at Notting Hill Carnival

By Hilary Lamb

Published Friday, September 1, 2017

A trial of facial recognition technology by police monitoring the Notting

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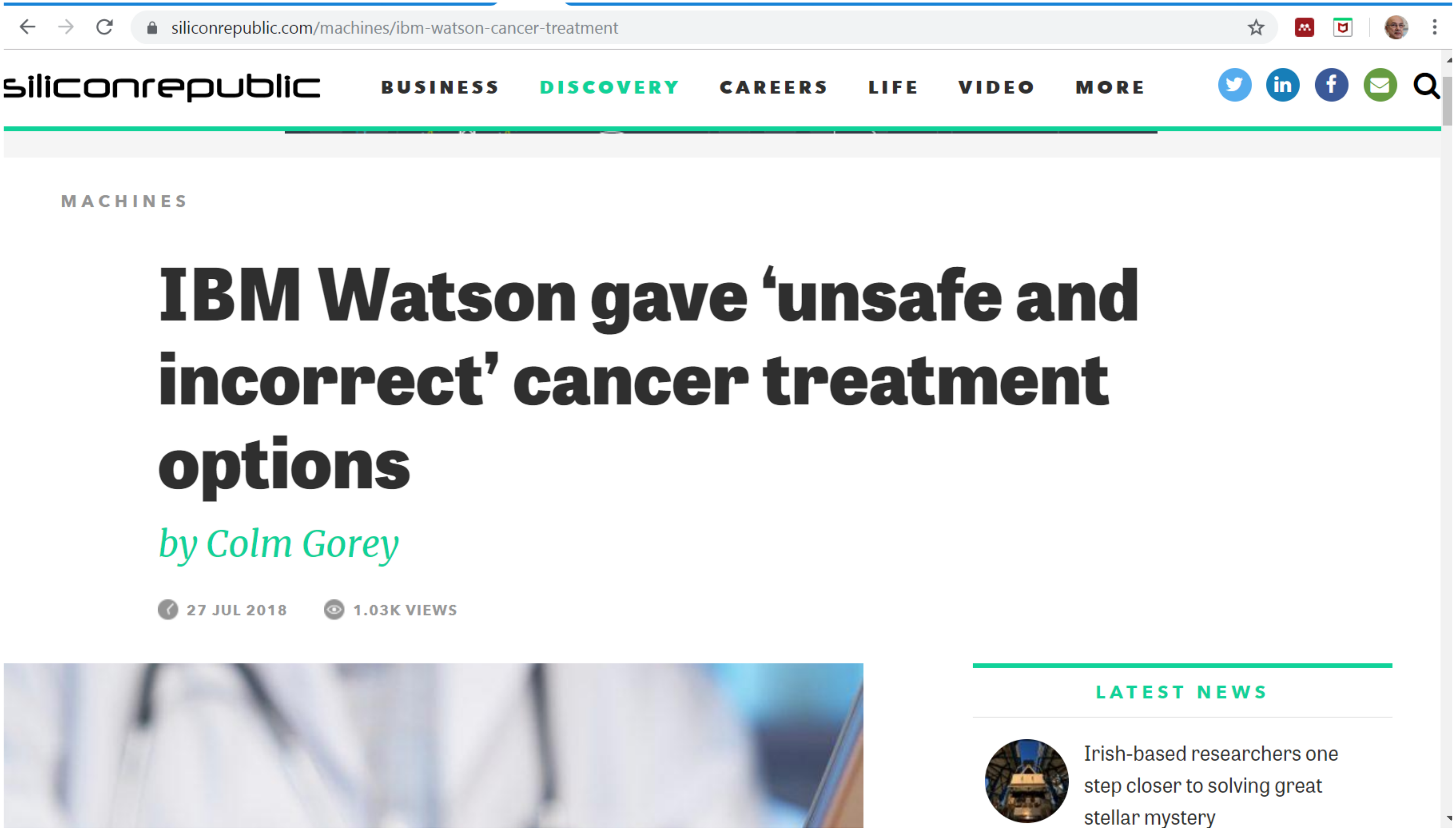

siliconrepublic.com/machines/ibm-watson-cancer-treatment

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MACHINES

IBM Watson gave 'unsafe and incorrect' cancer treatment options

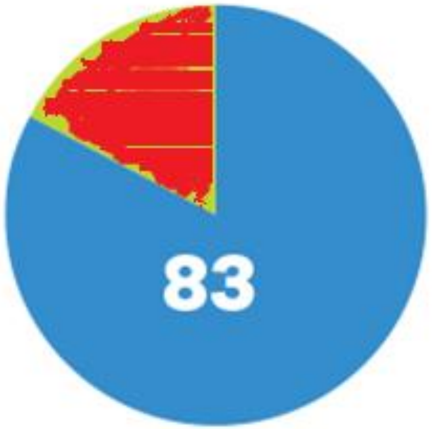
by Colm Gorey

27 JUL 2018 1.03K VIEWS

LATEST NEWS

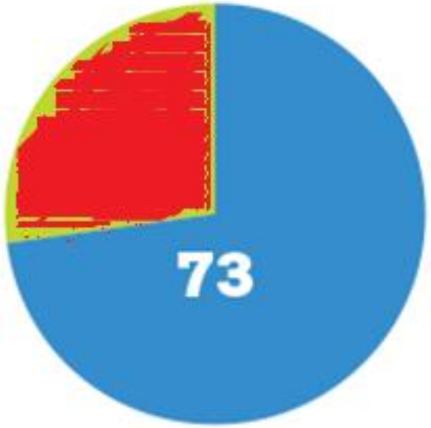
Irish-based researchers one step closer to solving great stellar mystery

Bumrungrad International Hospital, Thailand: 83% concordance



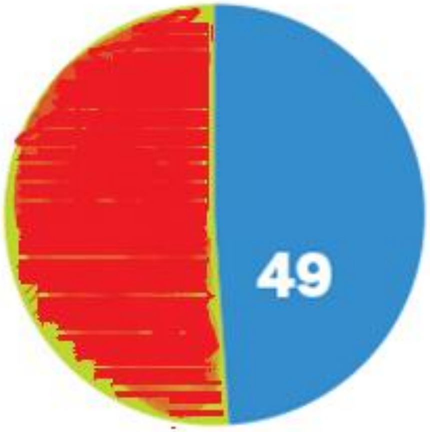
211 patients with breast, colorectal, gastric, and lung cancer

Manipal Comprehensive Cancer Center, India: 73% concordance



638 patients with breast cancer

Gachon University Gil Medical Center, South Korea: 49% concordance



656 patients with colon cancer

A 2016 audit by the University of Texas found that
the cancer centre spent $62 million on the project before canceling it.

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AI Errors: and the list continues ...

- **InsiperoBot** Poor Testing/Coding
- **Cortana** Not Working Bias/Limited Training Set Poor Testing/Coding
- **Alexa** Blasting Music Unexpected Human Behaviour
- **Passport Checker** Bias/Limited Training Set Poor Testing/Coding
- **Knightscope Robot** Hits Toddler Poor Testing/Coding Unexpected Human Behaviour
- **Facebook Translate** Arrest, Bias/Limited Training Set Unexpected Human Behaviour
- **Google Tag** Racist Bias/Limited Training Set
- **WeChat** Racist Bias/Limited Training Set Unexpected Human Behaviour
- **Microsoft - Tay** Unexpected Human Behaviour Bias/Limited Training Set

……………………………

(Thanks to Rosie Fenwick and Eliyas Woldegeorgis for collecting the list)

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Fundamental origins of AI errors

- Software errors
- Unexpected human behaviour
- Non-intended use
- ….
- **Uncertainty in training data (100 attributes require $> 2^{100} > 10^{30}$ records for completeness)**
- **Uncertainty in the training process**

Errors are unavoidable companions of data driven AI

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<u>Errors are unavoidable companions of data driven AI</u>

Message 1

The main AI 2020 problem:
Errors of AI and their processing

Errors are unavoidable companions of data driven AI

A new AI winter will come if we don't focus on the problem of AI errors

And only those applications will survive where errors are not very dangerous

Can we correct errors by systematic re-training?

- To preserve existing skills we must use the full set of training data.
- This approach requires much recourses for each error.
- However, new errors may appear.
- The preservation of existing skills is not guaranteed.
- The probability of damage is a priori unknown.

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Why?

D. Donoho, from Stanford University webpage

High-dimensional post-classical world

- **The number of attributes *p* >> The number of examples *N***
- This *post-classical* world is different from the '*classical world*'.
- The classical methodology was developed for the 'classical world' based on the assumption of $p < N$, and $N \to \infty$.
- These results all fail if $p > N$.
- The $p > N$ case is not anomalous; it is the generic case.

Donoho, D.L. High-Dimensional Data Analysis: The Curses and Blessings of Dimensionality. Invited Lecture at Mathematical Challenges of the 21st Century, AMS.

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Thus, the classical statistical learning theory is mostly useless in the multidimensional post-classical world.

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High-dimensional post-classical world

Comment 1

- All the geometric blessing and curse of dimensionality effects appear already when

$$p \gg \log N,$$

- That is, the “post-classical world” effects begin long before $p > N$.
- The non-classical world is around!

AN Gorban, VA Makarov, IY Tyukin, **High-Dimensional Brain in a High-Dimensional World: Blessing of Dimensionality** *Entropy* 22 (1), 82, https://doi.org/10.3390/e22010082

High-dimensional post-classical world

Comment 2

- The number of attributes differs significantly from the dimensionality of data.
- The definition of the “post-classical world” should be modified to

$$\mathrm{Dim(DataSet)} \gg N,$$

- or, according to Comment 1,

$$\mathbf{Dim(DataSet)} \gg \mathbf{log}\ \boldsymbol{N}.$$

About different definitions of data dimensionality related to blessing of dimensionality:
Bac, J., Zinovyev, A. (2020). **Lizard brain: tackling locally low-dimensional yet globally complex organization of multi-dimensional datasets**. *Frontiers in Neurorobotics*, *13*, 110. https://doi.org/10.3389/fnbot.2019.00110

How to cope with the non-classical world?

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Return to classics?

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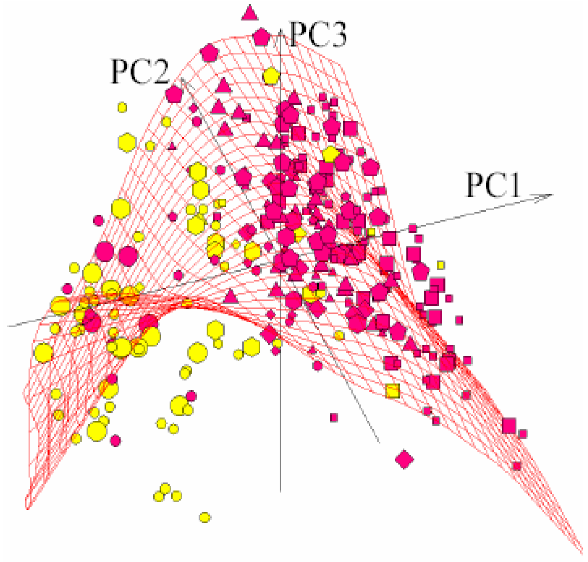
Return to classics:

- Reduce dimensionality
  (PCA, ICA, Principal graphs and manifolds, Autoencoders,… - methods of the previous century with some modern improvements);



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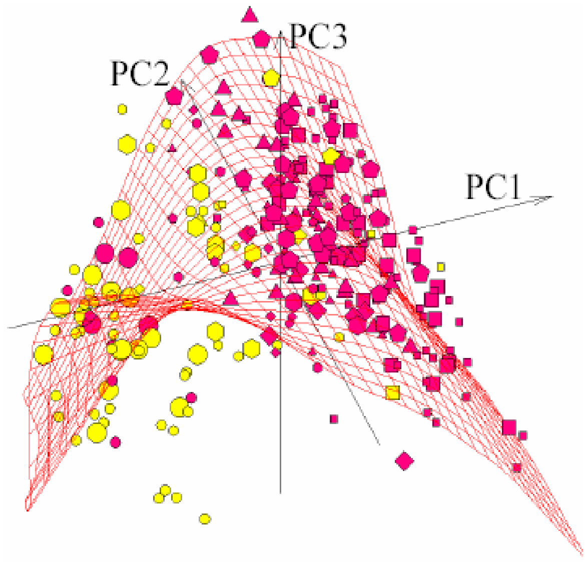
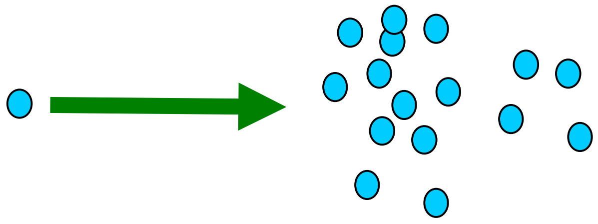


- Data augmentation – smearing data by random perturbations
  (also well-known from 1990s).



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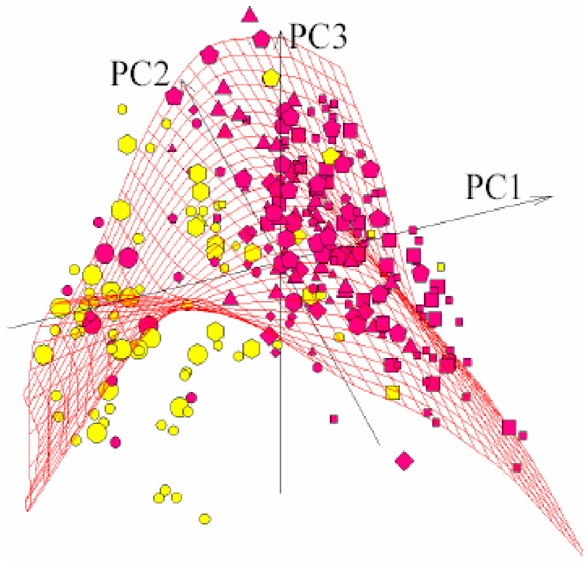

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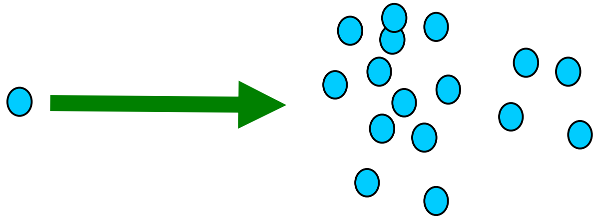

Sometimes this works: A. Gorban, B. Kégl, D. Wunsch, A. Zinovyev (Eds.), ***Principal manifolds for data visualization and dimension reduction.*** Springer, (2008). Gorban, A. N., Zinovyev, A. **Principal manifolds and graphs in practice: from molecular biology to dynamical systems**. *Int. J. Neural Syst.*, *20* (2010), 219-232.

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An *external device* - Corrector of AIs' mistakes is needed

A.N. Gorban, I.Y. Tyukin, Blessing of dimensionality: mathematical foundations of the statistical physics of data. *Philos. Trans. Royal Soc.* A 376(2118), 20170237, 2018.

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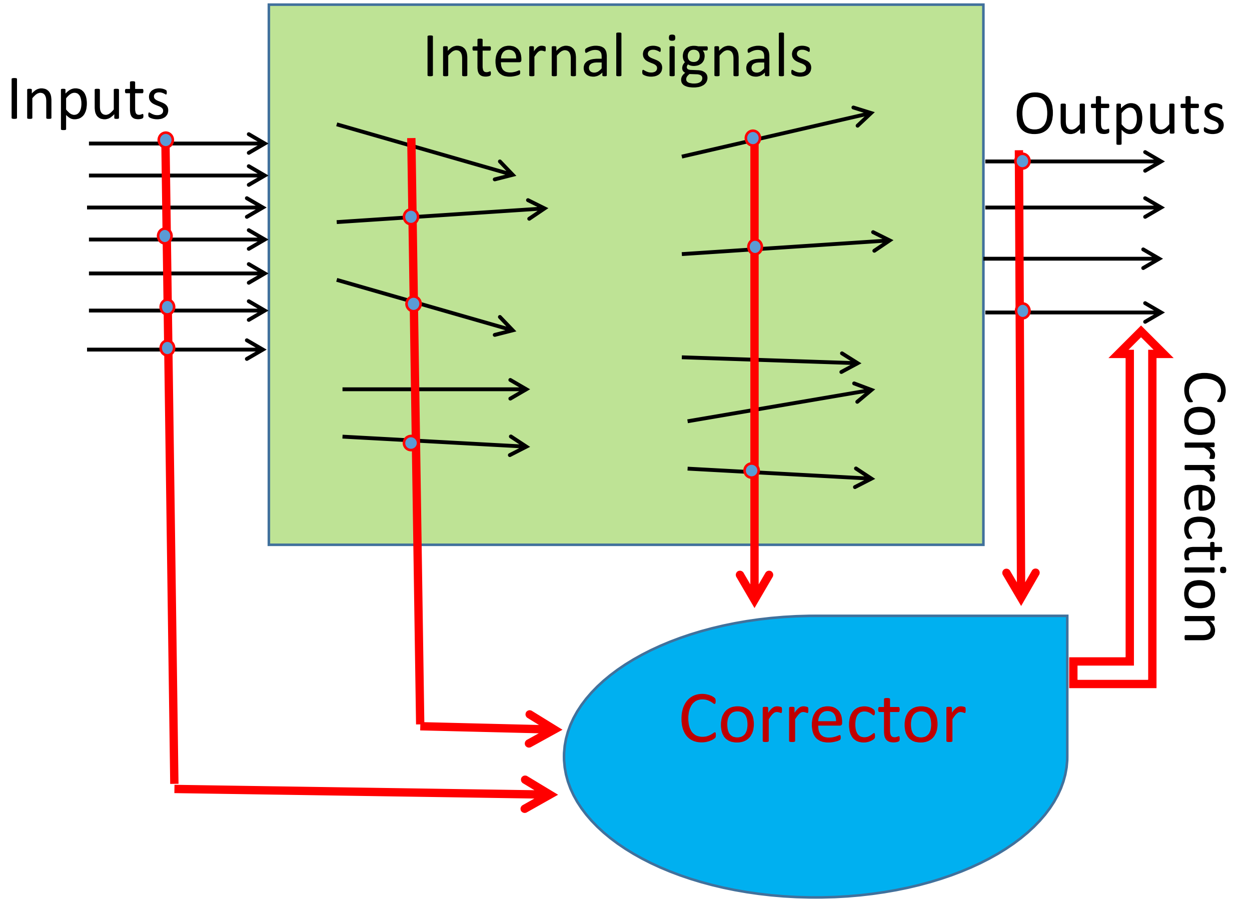
Corrector has to separate mistakes from correctly solved examples and correct them



Corrector is a **binary classifier** *for error diagnosis* equipped by the **modified decision rule** *for high risk situations*.

Gorban, A. N., Golubkov, A., Grechuk, B., Mirkes, E. M., Tyukin, I. Y. [Correction of AI systems by linear discriminants: Probabilistic foundations](). *Information Sciences*, *466* (2018), 303-322.

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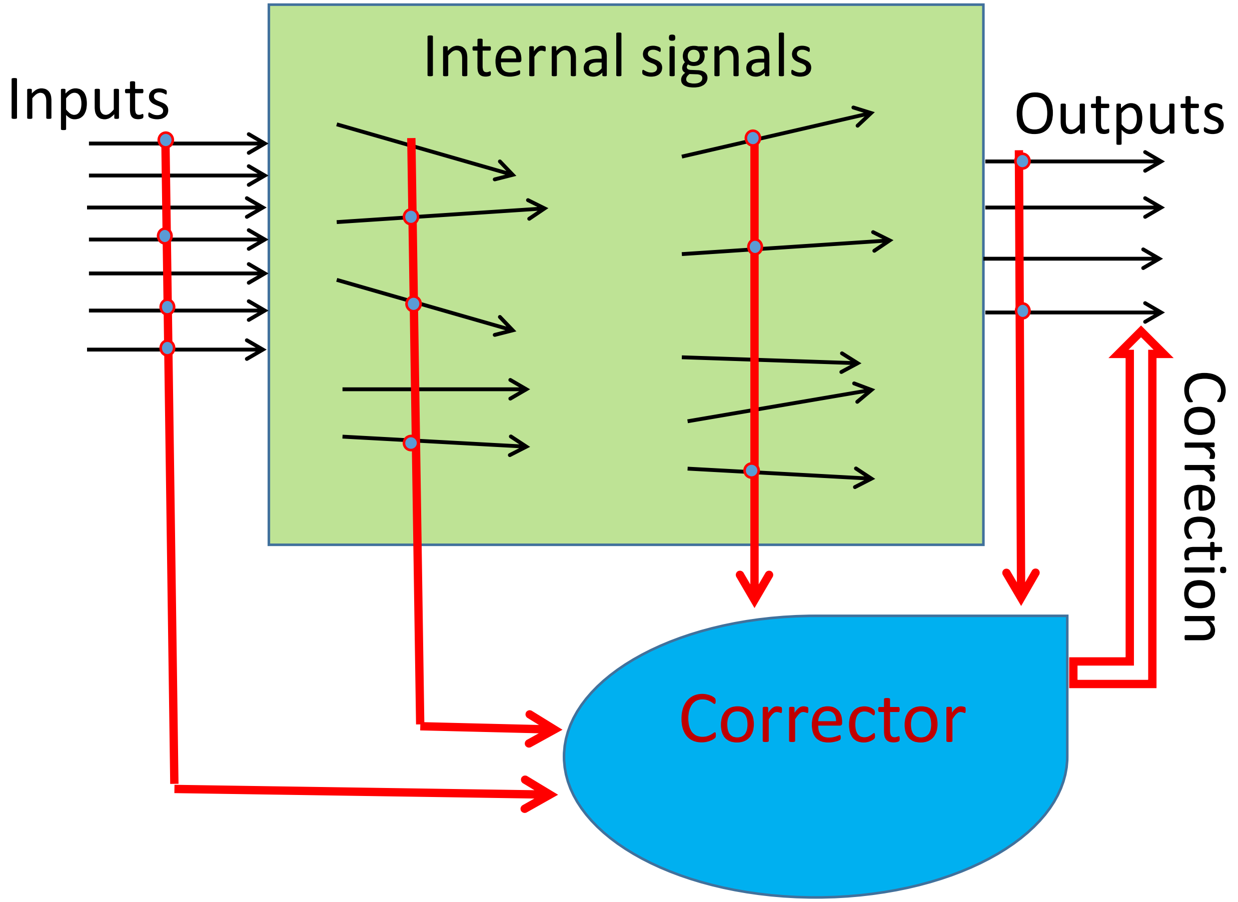

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Fisher's linear discriminants separate errors from other examples surprisingly well in high dimensions.

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Fisher discriminants are explicit and non-iterative classifiers

Let a data set *Y be centralized* (zero mean) and *whitened* (unit covariance matrix)

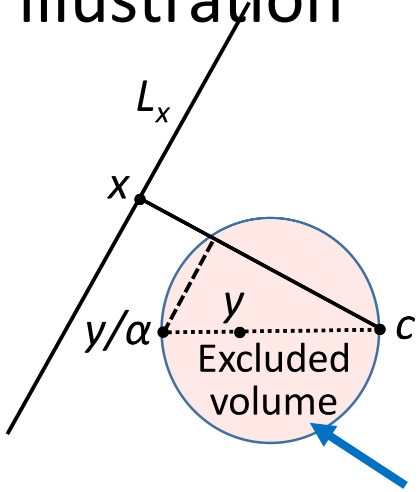
Definition. *A point x is Fisher separable from a finite set Y with a threshold α (0 ≤ α < 1) if*

$$(x,y) \le \alpha (x,x) \qquad (1)$$

for all y from Y.

Where (x,y) is the standard inner product.

(1) illustration



x does not belong to a ball

$$\left\{ z \;\middle|\; \left\| z - \frac{y}{2\alpha} \right\| < \frac{\|y\|}{2\alpha} \right\}$$

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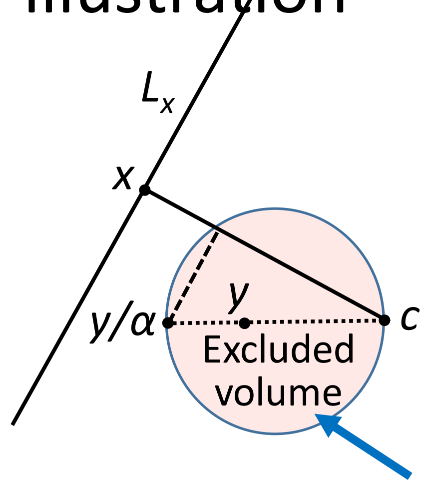

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$$\left\{ z \,\middle|\, \left\| z - \frac{\mathbf{y}}{2\alpha} \right\| < \frac{\|\mathbf{y}\|}{2\alpha} \right\}$$

These simple discriminants effectively separate mistakes from correctly solved examples in high dimensions.

Gorban, A. N., Burton, R., Romanenko, I., & Tyukin, I. Y. (2019). One-trial correction of legacy AI systems and stochastic separation theorems. *Information Sciences*, *484*, 237-254.

A miracle of stochastic separation

(an old joke about a new topic)

Let us be a bit more explicit!

Gorban, A. N., Tyukin, I. Y. (2017). **Stochastic separation theorems**. *Neural Networks*, *94*, 255-259.

Assume the absence of: (i) large deviations and (ii) sets with small volume but high probability

Theorem 1. *Assume that a probability distribution in the n-dimensional unit ball $\mathbb{B}_n$ (with volume $V_n(\mathbb{B}_n)$) has a density with maximal value ρ_m, which satisfies inequality*

$$\rho_m < \frac{C}{r^n V_n(\mathbb{B}_n)} \quad 0 < r \leq 1.$$

Let $|Y| < b^n$ and $2r\alpha > b > 1$. Then the probability p that a random point is Fisher-separable from the finite set Y is $p = 1 - \psi$, where

$$\psi < C\left(\frac{b}{2r\alpha}\right)^n.$$

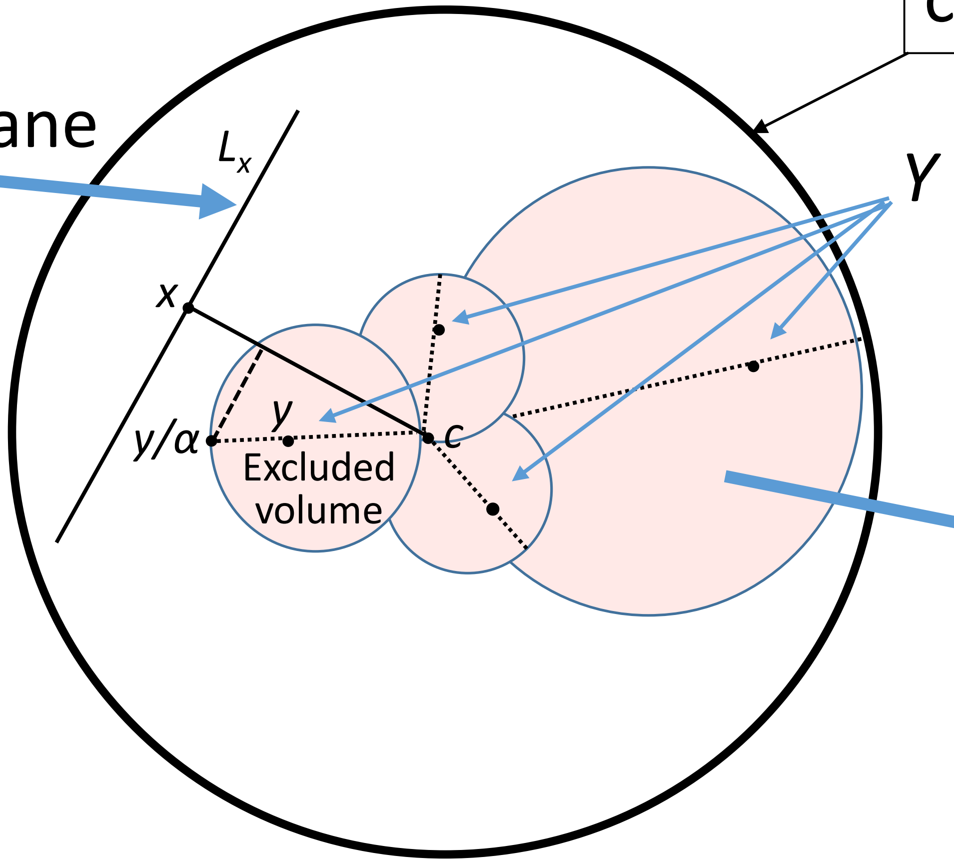
Note: $r \leq 1$, $\alpha < 1$
but $r\alpha > 0.5$

Gorban, A. N., Golubkov, A., Grechuk, B., Mirkes, E. M., Tyukin, I. Y. Correction of AI systems by linear discriminants: Probabilistic foundations. *Information Sciences*, *466* (2018), 303-322.

For proof: just evaluate excluded volumes and probability of *x* to be there

Unit ball with centre *c*

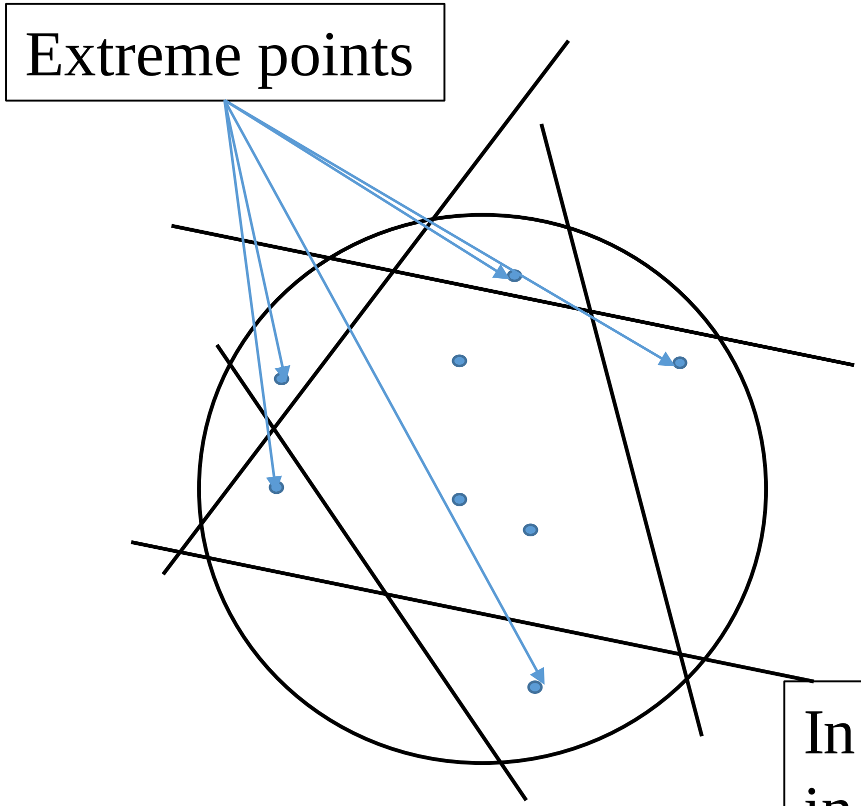
The hyperplane of Fisher's discriminant



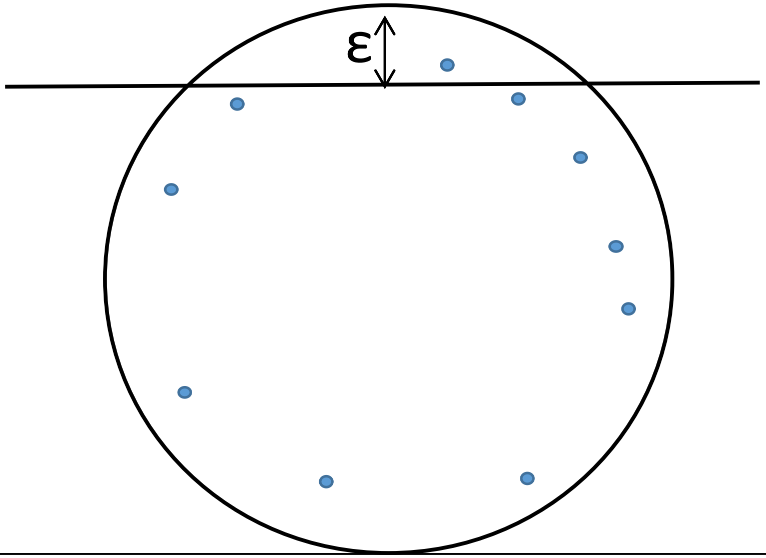
Volume of one "excluded ball" is less than

$$V_n(\mathbb{B}_n)/(2\alpha)^n,\ \ 2\alpha>1$$

Extreme points in high dimensions

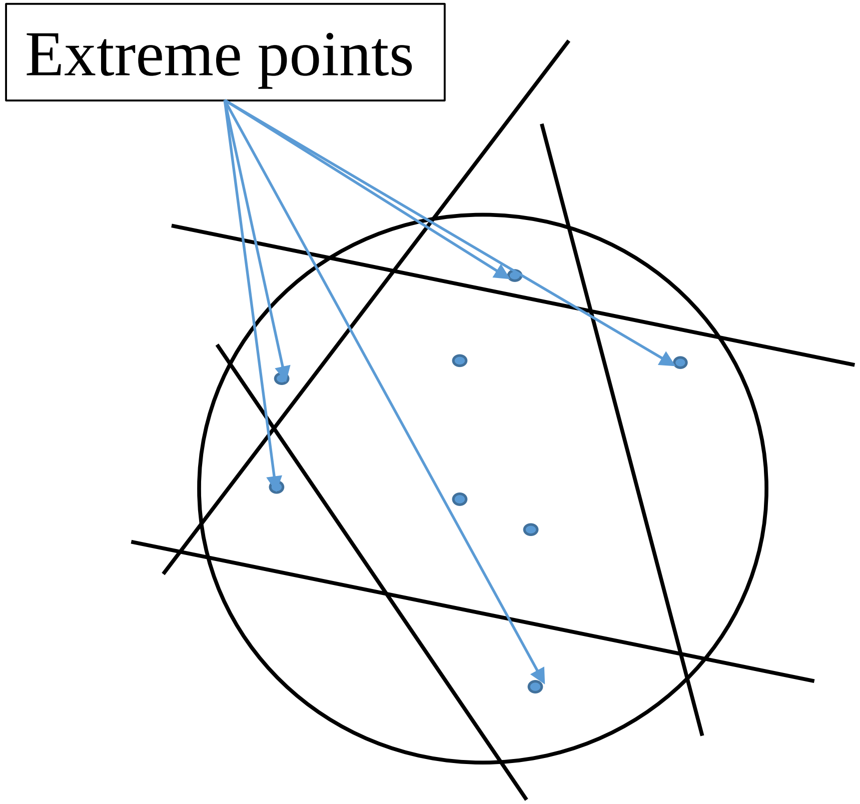


Expected ε IS NOT small
in high dimension!



In high dimension, with high probability even in an exponentially large random set all points are extreme ones!

Extreme points in high dimensions



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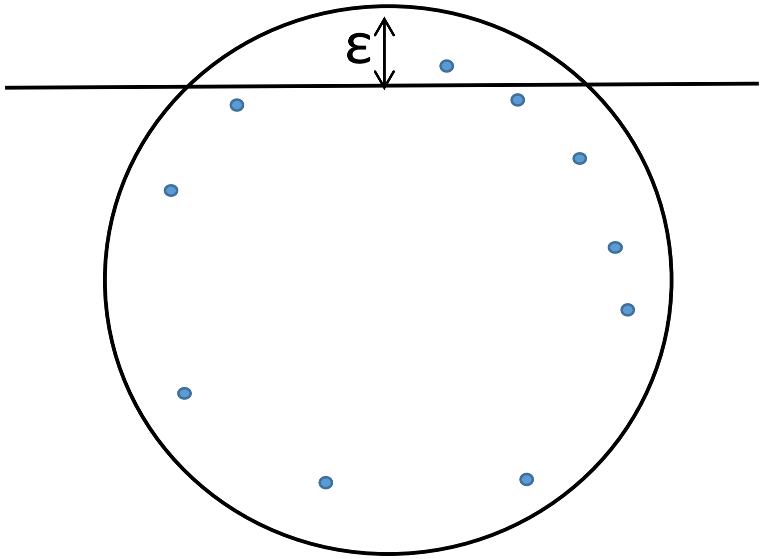

<u>*Example:*</u>

For ***n*=100** and ***M* < 2,740,000** the set of *M random* points in a ***n***-dimensional ball is Fisher-separable (each point from the rest) with probability ***p*>99%**

Blessing of dimensionality

ELEMENTARY PRINCIPLES

IN

STATISTICAL MECHANICS

DEVELOPED WITH ESPECIAL REFERENCE TO

THE RATIONAL FOUNDATION OF THERMODYNAMICS

BY

J. WILLARD GIBBS

Professor of Mathematical Physics in Yale University

NEW YORK: CHARLES SCRIBNER'S SONS
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avec un complément
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1951

 70

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- The blessing of dimensionality is a manifestation of the ***concentration of measure.***

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Blessing of dimensionality

- The blessing of dimensionality is a manifestation of the *concentration of measure.*
- All statistical physics is based on these phenomena.
- **Stochastic separation theorems represent a new class of them.**

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Plenty of generalizations for distributions

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Plenty of generalizations for distributions

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- Product distributions with bounded support;
- **No i.i.d. assumptions.**

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Heuristic. **The stochastic separation theorems hold if:**

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Heuristic: The stochastic separation theorems hold if:

- **There are no heavy tails of the probability distribution;**

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- There are no heavy tails of the probability distribution;
- **The sets of small volume should not have large probability** (what “small” and “large” mean is strictly defined for different contexts).

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Heuristic: The stochastic separation theorems hold if:

- There are no heavy tails of the probability distribution;
- The sets of small volume should not have large probability
  (what “small” and “large” mean is strictly defined for different contexts).

In these cases, Fisher discriminant is an effective tool for classification and AI correctors in high dimension

Plenty of generalizations for classifiers

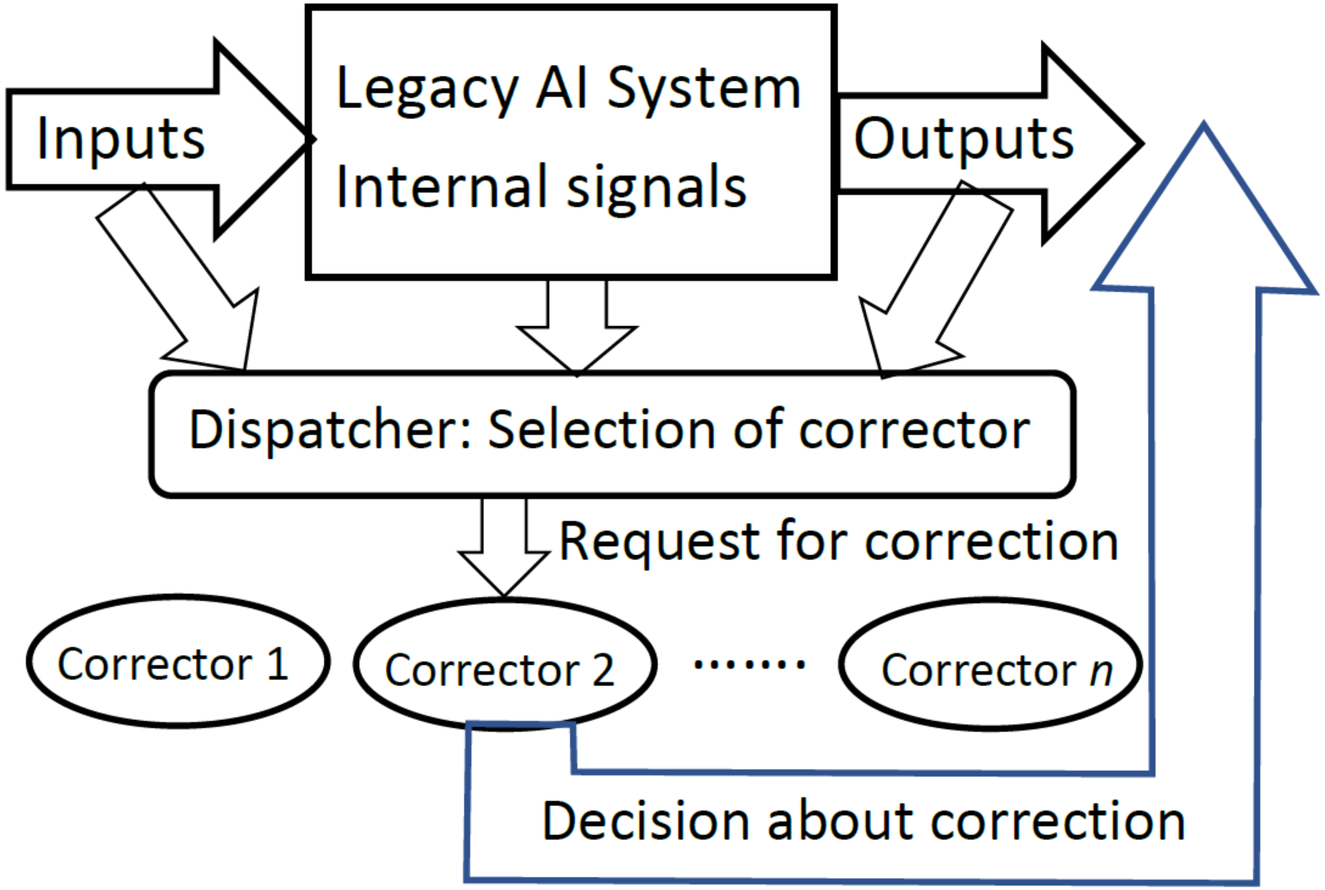
Plenty of generalizations for classifiers

- **Kernel classifiers**

Tyukin, I. Y., Gorban, A. N., Grechuk, B., Green, S. (2019, July). **Kernel Stochastic Separation Theorems and Separability Characterizations of Kernel Classifiers.** In *2019 International Joint Conference on Neural Networks (IJCNN)* (pp. 1-6). IEEE.

Plenty of generalizations for classifiers

- Kernel classifiers;
- **Ensembles of correctors (with clustering of errors);**



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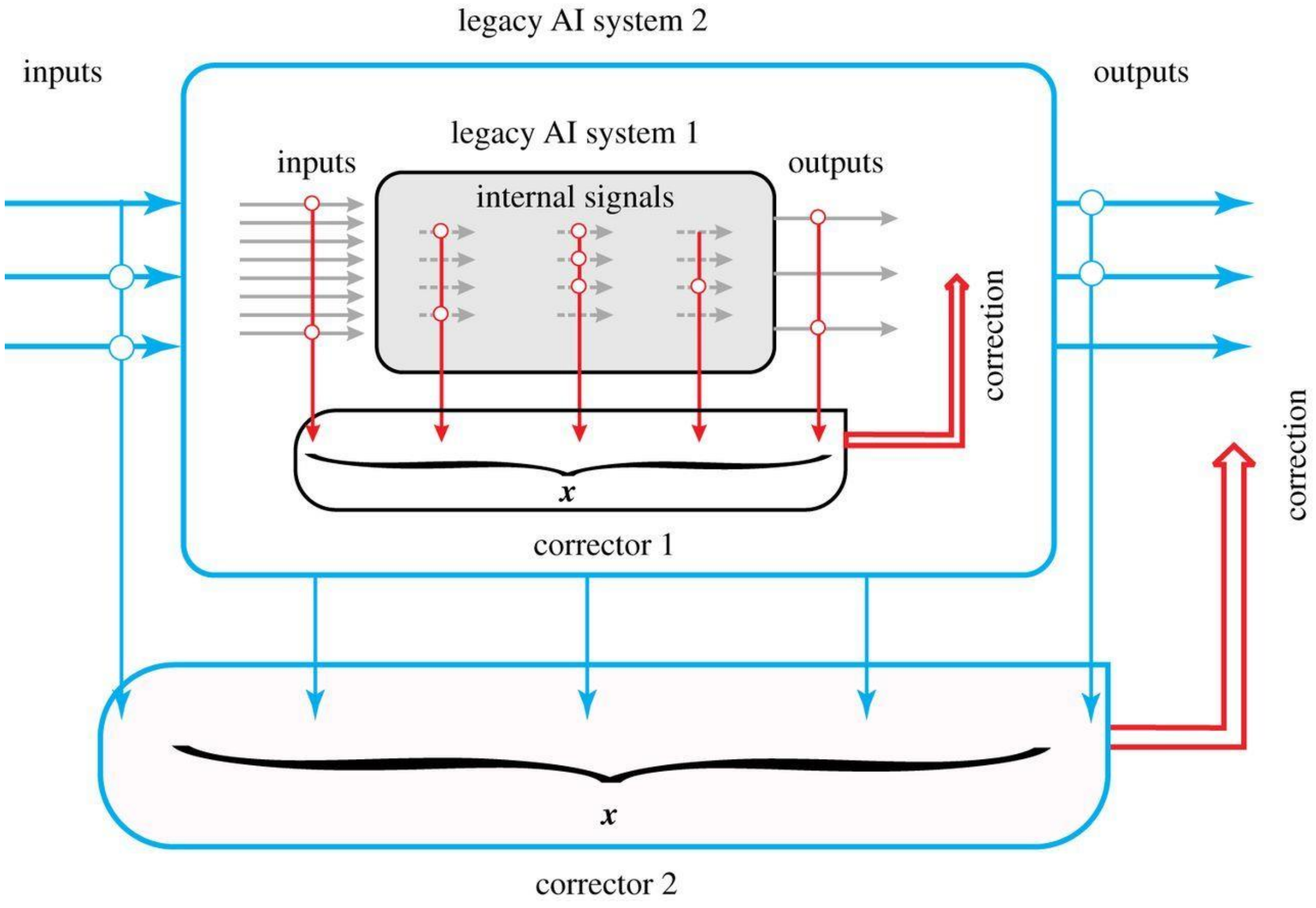
- Kernel classifiers;
- Ensembles of correctors (with clustering of errors);
- **Cascades of correctors.**



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Interiorisation of knowledge

- **Complex correctors correct many errors.**

Sculley, D., Holt, G., Golovin, D.... & Dennison, D. (2015). Hidden technical debt in machine learning systems. In *Advances in neural information processing systems* (pp. 2503-2511).

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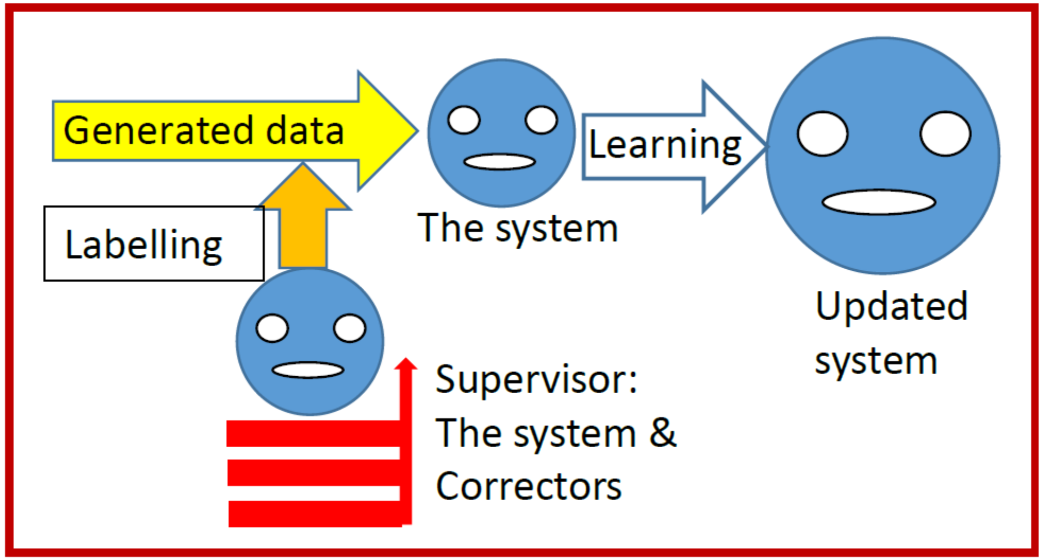
Knowledge interiorisation



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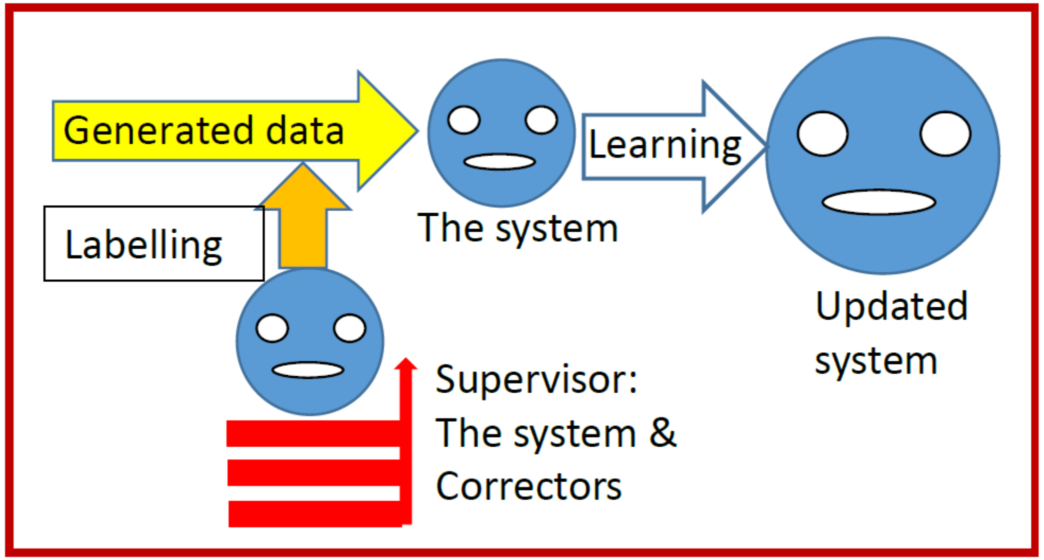

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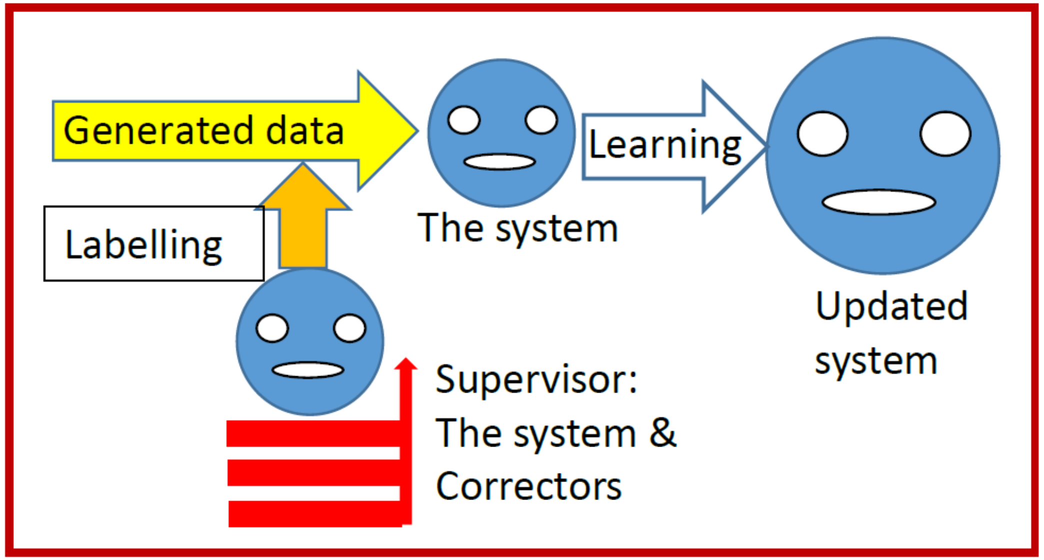

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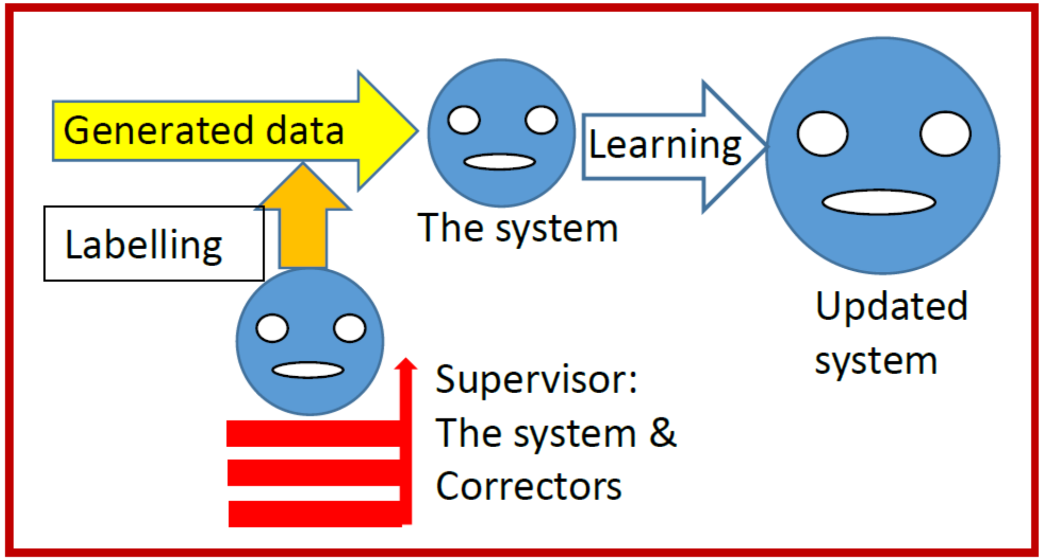

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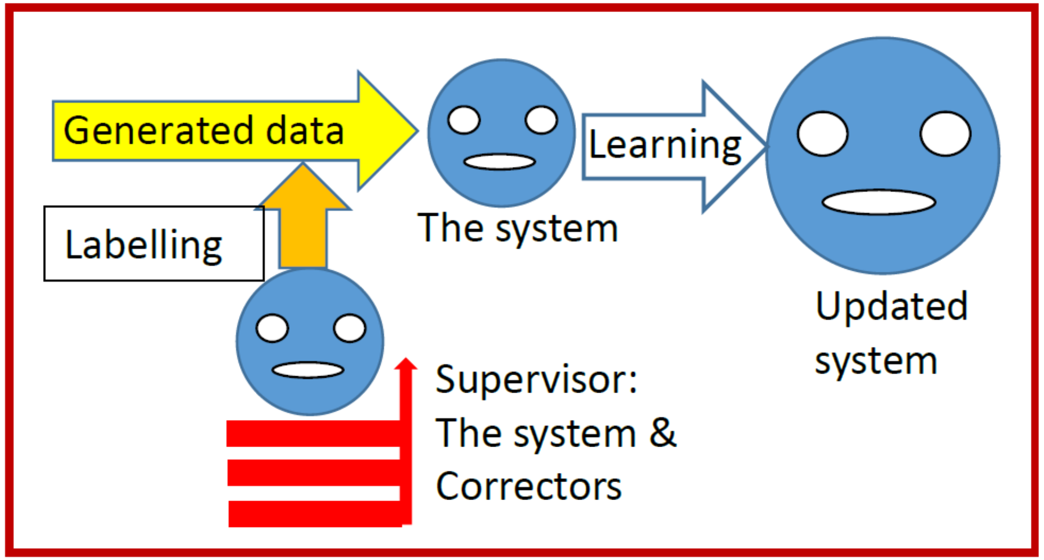
- **Interiorisation can be organised as supervised learning that uses the system with correctors as the supervisor.**

Knowledge interiorisation



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Adversarial Examples and Stealth Attacks in Artificial Intelligence Systems

Tyukin, I. Y., Higham, D. J., & Gorban, A. N. (2020). **On Adversarial Examples and Stealth Attacks in Artificial Intelligence Systems**. *arXiv:2004.04479*.

Adversarial Examples and Stealth Attacks in Artificial Intelligence Systems

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- **Robustness to adversarial examples requires either**
  (a) the data distributions in the AI’s feature space to have concentrated probability density functions or
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- Robustness to adversarial examples requires either
  (a) the data distributions in the AI's feature space to have concentrated probability density functions or
  (b) the dimensionality of the AI's decision variables to be sufficiently small.
- **The stealth attacks on high-dimensional AI systems are discovered that are hard to spot unless the validation set is made *exponentially large*.**

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Message 2: Non-classical alternative

In the non-classical world we have a choice:
either return to classics,
or exploit the blessing of dimensionality
and create one-shot correctors of errors.

When AI can play with AI then the progress is much faster

- Correctors are tools for one-shot transferring skills between AIs.

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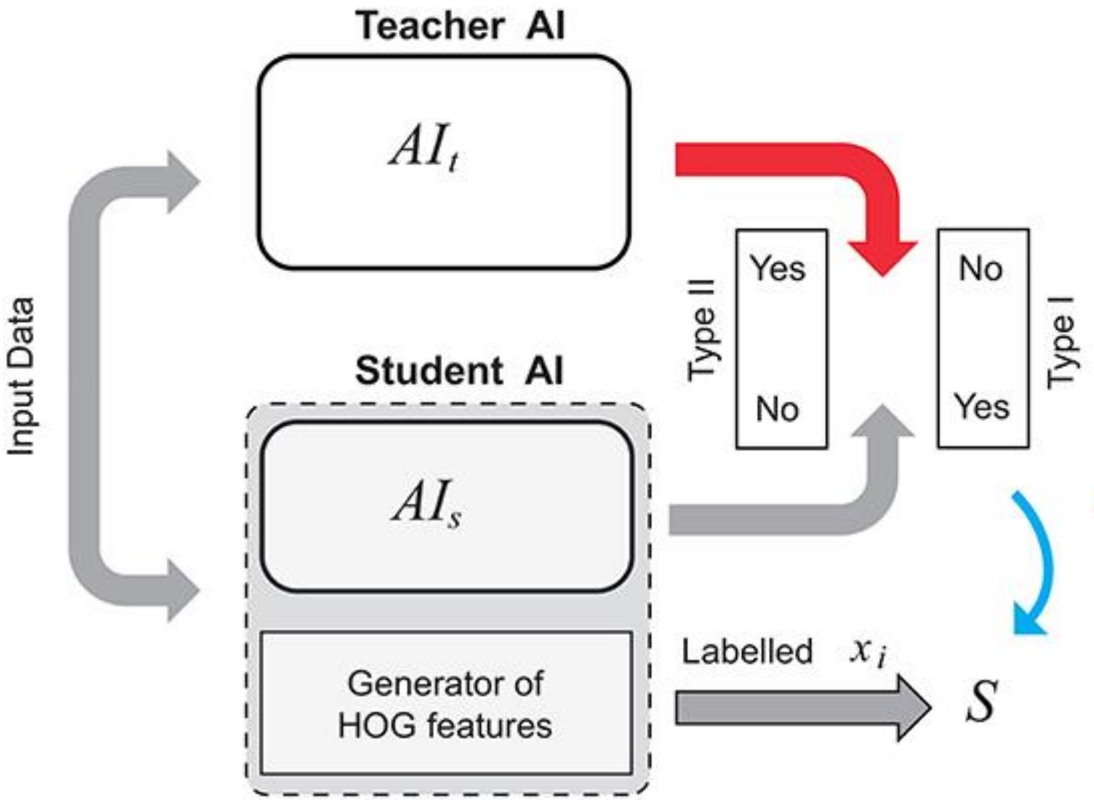
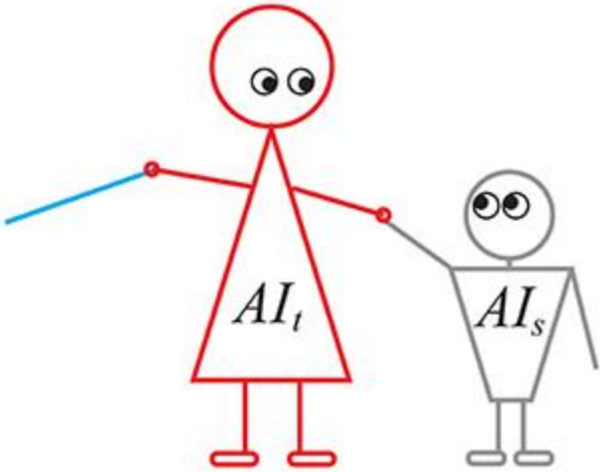
- Correctors are tools for one-shot transferring skills between AIs.
- Experiment: training of pedestrian detection



Tyukin, I. Y., Gorban, A. N., Sofeykov, K. I., Romanenko, I. (2018). [Knowledge transfer between artificial intelligence systems](). *Frontiers in neurorobotics*, *12*, 49.



Multiagent neuro-AI systems

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- For each problem, the agent can evaluate his level of *confidence.*
- They rather learn to be *not too confident in their mistakes* than not to make mistakes at all.
- **To successfully interact, agents should *cooperate, coordinate, conflict, negotiate, and correct* each other.**

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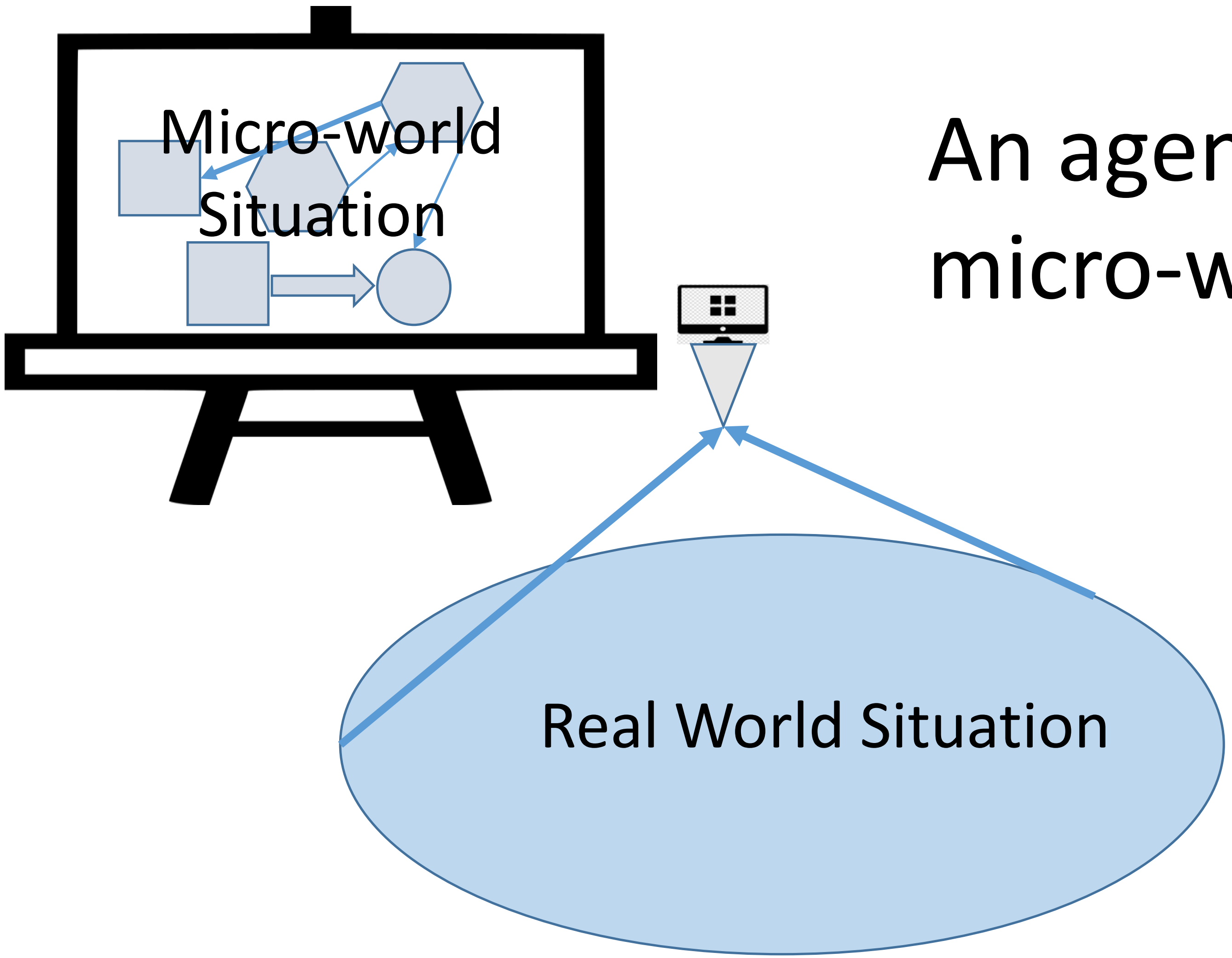
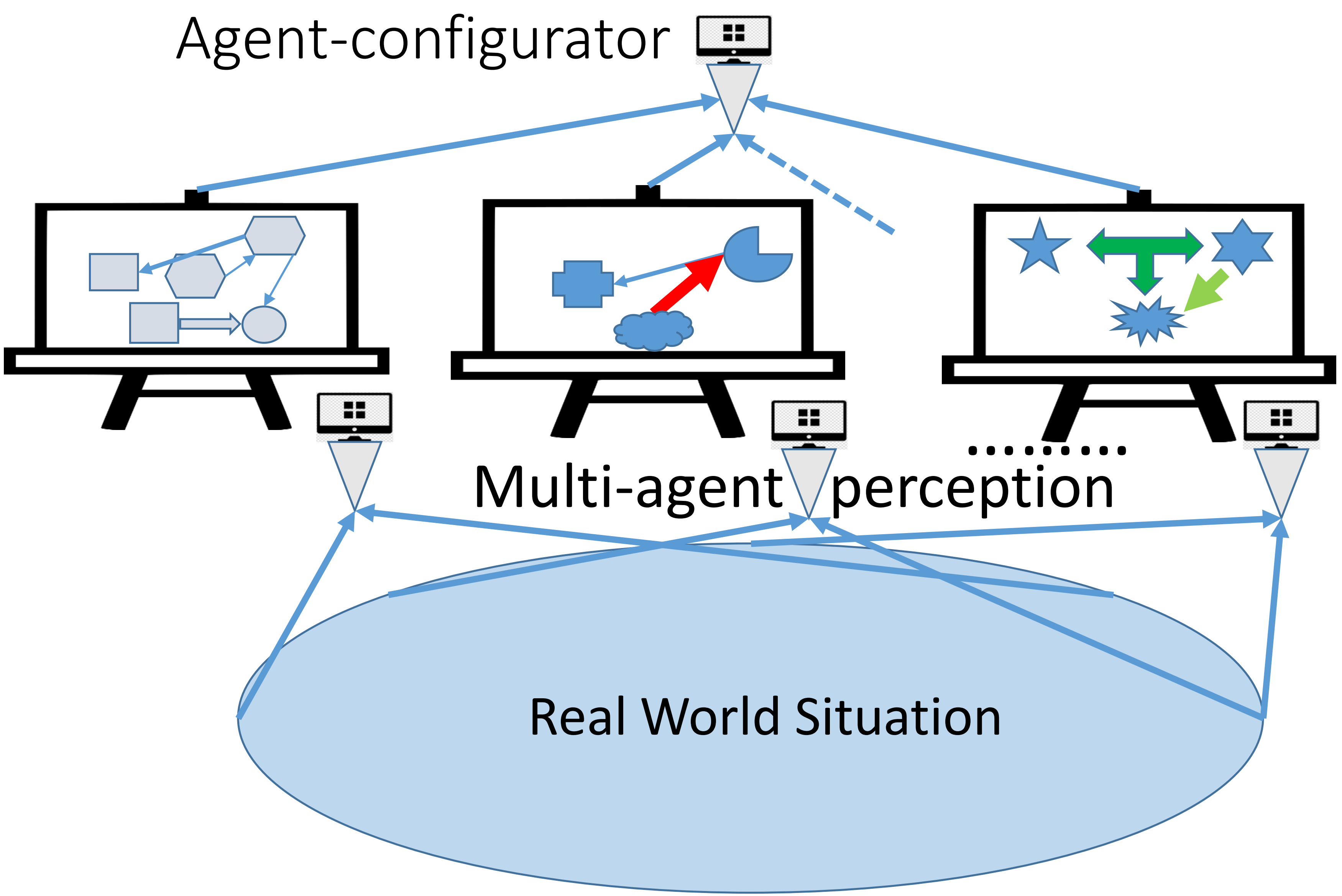
A personal micro-world is a working place for AI (M. Minsky)



An agent and his micro-world

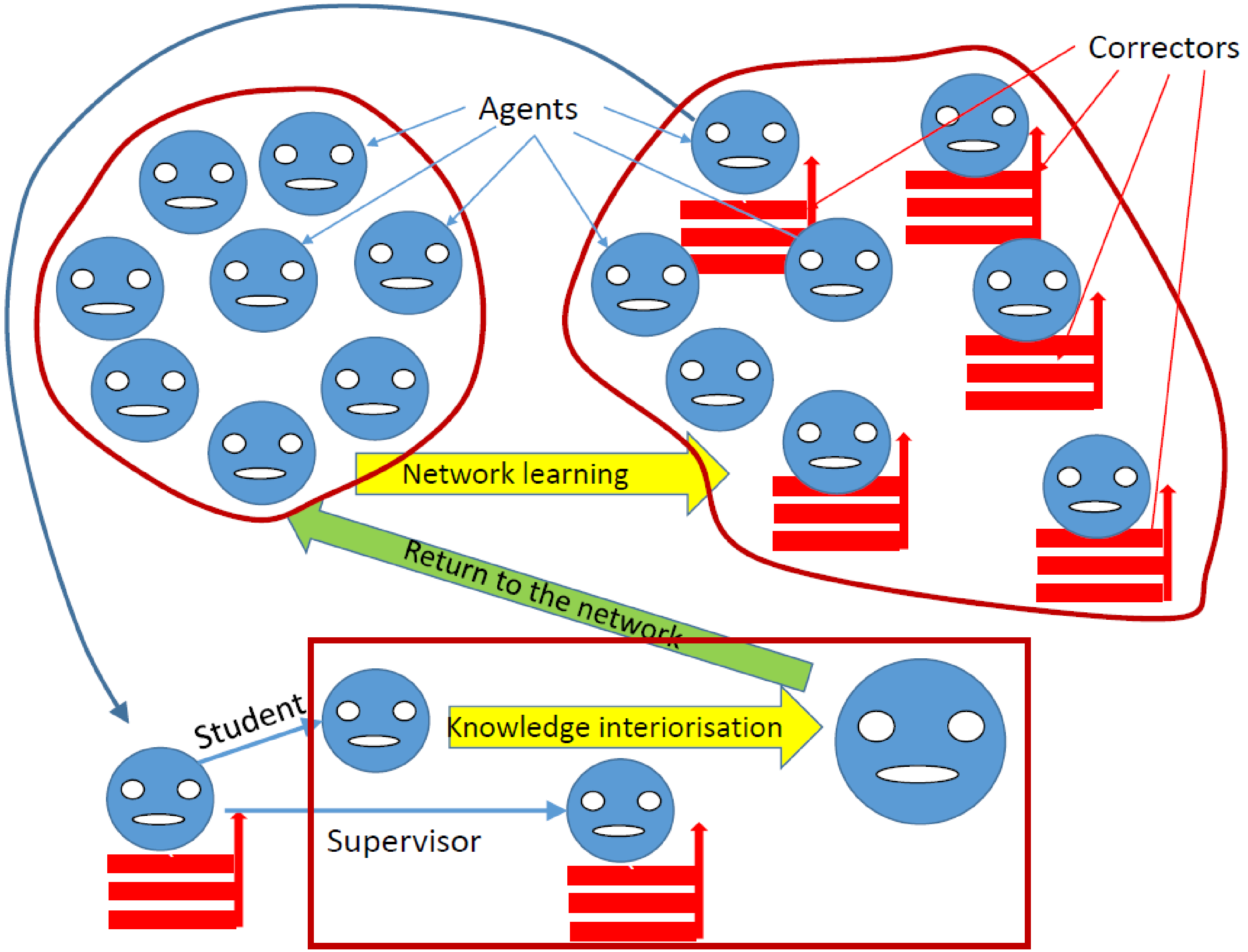
Multilayer multi-agent structures



Neuro-AIs in community

Stages of community learning:

1. Pre-learning;
2. Network learning with mutual corrections;
3. Interiorisation of correctors performed for agents independently;
4. Updating of community members.



Message 3: After tomorrow AI will be the large social system of heterogeneous AI networks

To meet the after tomorrow future
we have to move from the individual AI system
to engineering of the large ecosystems and social systems
of heterogeneous AI networks in heterogeneous high-dimensional world.

When explainability of AI is needed?

Decisions of Neural AI are not transparent and, therefore, cannot be explained logically.

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- **We developed software that produced "logically transparent" neural networks by pruning and learning since middle of 1990s.**

Mirkes, E. M. (2020). **Artificial Neural Network Pruning to Extract Knowledge**. *arXiv:2005.06284*.

When explainability of AI is needed?

Decisions of Neural AI are not transparent and, therefore, cannot be explained logically.

- We developed software that produced “logically transparent” neural networks by pruning and learning since middle of 1990s.
- **Experience: Users preferred “one-button” solutions and used this option rarely.**

Mirkes, E. M. (2020). **Artificial Neural Network Pruning to Extract Knowledge**. *arXiv:2005.06284*.

When explainability of AI is needed?

Decisions of Neural AI are not transparent and, therefore, cannot be explained logically.

- We developed software that produced “logically transparent” neural networks by pruning and learning since middle of 1990s.
- Experience: Users preferred “one-button” solutions and used this option rarely.
- **In most cases of use, the logically transparent form was needed to analyse errors.**

Mirkes, E. M. (2020). [Artificial Neural Network Pruning to Extract Knowledge](). *arXiv:2005.06284*.

Explainability of AI is needed when AI makes mistakes

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- **To correct the erroneous steps for the future.**

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- To correct the erroneous steps for the future.
- **Simplify. Credibility is simplicity.**

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- To decompose the erroneous decision into elementary steps;
- To understand the source of mistake: which step was wrong?
- To distribute responsibility for this mistake: who is responsible and for what?
- To correct the erroneous steps for the future.
- Simplify. Credibility is simplicity.
- **But… Does truth resists simplicity?**

Four messages

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1. **Errors are unavoidable companions of data driven AI: The main AI 2020 problem: Errors of AI and their processing**

Four messages

1. Errors are unavoidable companions of data driven AI: The main AI 2020 problem: Errors of AI and their processing
2. **Non-classical alternative: In the non-classical world we have a choice: either return to classics, or exploit the blessing of dimensionality and create one-shot correctors of errors.**

Four messages

1. Errors are unavoidable companions of data driven AI: The main AI 2020 problem: Errors of AI and their processing
2. Non-classical alternative: In the non-classical world we have a choice: either return to classics, or exploit the blessing of dimensionality and create one-shot correctors of errors.
3. **After tomorrow AI will be the large social system of heterogeneous AI networks.**

Four messages

1. Errors are unavoidable companions of data driven AI: The main AI 2020 problem: Errors of AI and their processing
2. Non-classical alternative: In the non-classical world we have a choice: either return to classics, or exploit the blessing of dimensionality and create one-shot correctors of errors.
3. After tomorrow AI will be the large social system of heterogeneous AI networks.
4. **Explainability of AI is needed when AI makes mistakes.**

A new AI winter will come if we don't focus on the problem of AI errors

And only those applications will survive where errors are not very dangerous

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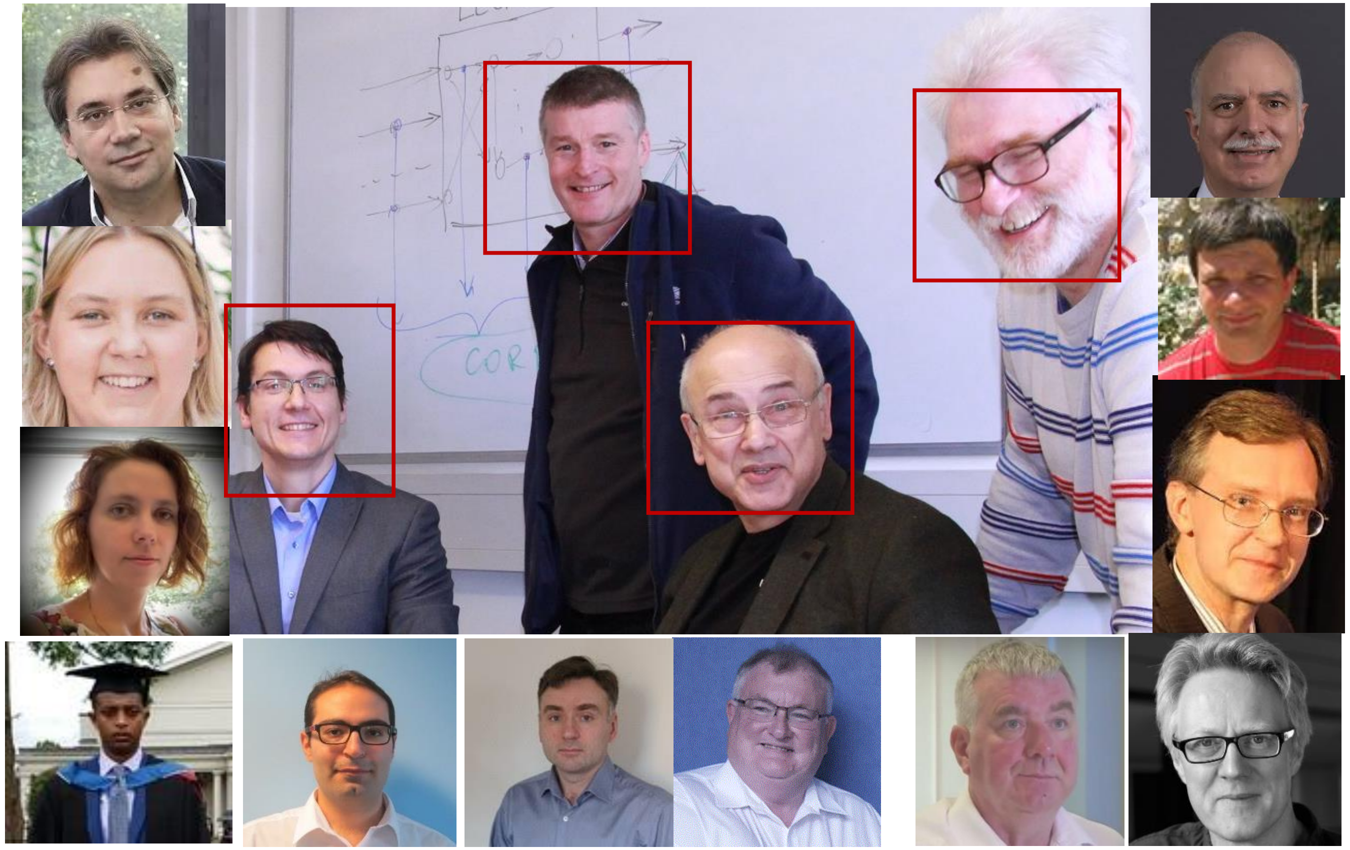

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Thank you for your attention

Questions please